Using a Catenary Equation in Parametric Representation for Minimizing Stress Concentrations at Notches

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1. Introduction

1.1 Introduction into the theme

State-of-the-art
• Usually, simple circular radius fillets are used to reduce stress concentrations at cross section transitions
• The stress concentration factor of these fillets can be simply read-out from diagrams from literature (e.g. [1]) for the given dimensions of the cross-section transition
• Anyway, the efficiency of the stress reduction is very limited for circular radius fillets

Non-circular notches
• are much better in reducing the stress concentration factor $K_t$
• But, until now non-circular notches cannot be computed as simple as circular ones
• Usually, the FEM method has to be used to individually compute and optimize these shapes
1. Introduction

1.1 Introduction into the theme

Improved non–circular notch design procedure
• In the last–year SAXSIM presentation shown below [2], the authors propose to design such non–circular notches by using special normalized diagrams
• These diagrams have been derived by using Creo Parametric as CAD– and Creo Simulate as p–FEM–tool
• Several different notch shapes have been examined:

- One radius fillet
- Two radii fillet
- Baud fillet
- Method of tensile triangles
- Standard elliptical fillet
- Conical round as generalized elliptical fillet
1. Introduction

1.1 Introduction into the theme

Basic procedure to derive a design diagram for a non–circular notch shape [2]:

• Parameterize the notch by using meaningful parameters within the CAD–tool (Creo Parametric)
• Vary these parameters in global sensitivity studies of the embedded p–FEM–tool (Creo Simulate)
• Draw the diagram for a normalized set of parameters

Limitation in deriving diagrams for non–circular notch shapes

• If the notch geometry cannot be drawn by the internal set of geometry creation features of the CAD system, the notch curve has to be coded in the equation editor
• The only way to do this within Creo Parametric is to transform the curve equation into a parametric representation, since this is the input format the equation editor expects from the user

Goal

• Transform the example curve (a catenary) into parametric representation, e.g. by using Mathcad (→ see associated presentation of Dr. Wigand Rathmann, [3])
• Code the curve in the equation editor of Creo Parametric and do all the subsequent examinations – sensitivity and optimization studies– within Creo Simulate
1. Introduction

1.2 Normalized model of the cross section change used for notch analysis [2]

Varying notch geometry

Exemplary material: Aluminum
Poisson’s ratio $\nu = 0.3$
E-modulus $E = 70$ GPa

$D = 200$ mm
$H = 200$ mm
$l = 100$ mm
$d = 10$ mm
$\sigma_{nom} = 100$ MPa

Tensile force

Normalization by the web thickness $d$: $d^* = d/d = 1$

$H^* = H/d = 20$

$D/2 = 100$ mm
$D^* = D/d = 20$

$D = 200$ mm
2. Computing the Ideal Catenary Notch

2.1 The example notch – a catenary curve

Catenary equation
- Catenaries are often used in civil engineering
- Goal is to check if it is also useful for an improved notch design

Catenary equation
- The usual form of the catenary curve is as follows:

\[ y = a \cdot \cosh\left(\frac{x}{a}\right) = \frac{a}{2} \left( e^{x/a} + e^{-x/a} \right) \]

Catenary arches under the roof of Gaudí’s Casa Milà, Barcelona, Spain (from Wikipedia)
2. Computing the Ideal Catenary Notch

2.1 The example notch – a catenary curve

Transfer into the required parametric equation
Some requirements have to be taken into account:

- A slight rotation of the catenary is necessary to have a tangent transition to the tension loaded web (otherwise, this is only assured if the notch height approaches infinity)
- If possible, good manageable parameters like notch height $h$ and width $b$, also to obtain a stable sketch for optimization
- If possible, fixed boundaries for the control variable $t$ of the parametric representation (e.g. from 0...1, since boundaries can not be parameterized in Creo Parametric!)

- Transfer into the required parametric representation now shown by Dr. Wigand Rathmann [3]
2. Computing the Ideal Catenary Notch

2.2 Coding the catenary with Creo Parametric functionality

Parameters

- $h$ for notch height and $t_1$ (or $abst_1$, respectively) for the starting point on the unscaled catenary curve
- Some local parameters for simplified coding
- Control variable $t$ is always between 0...1!

Measure notch width $b$ (as function of parameter $t_1$) for simple drawing of diagrams!
2. Computing the Ideal Catenary Notch

2.3 Analysis of the catenary in Creo Simulate

• Definition of a linear static analysis as 2D plain stress in advanced SPA with strongly increased accuracy
2. Computing the Ideal Catenary Notch

2.3 Analysis of the catenary in Creo Simulate

- Global Sensitivity Studies for drawing parametric diagrams
- Note: Parameter values for t1 (abst1, respectively) are always the same for the same h/b-value!
- Examined relation h/b=1…12
2. Computing the Ideal Catenary Notch

2.4 Results of the catenary analysis

Stress concentration factor $K_f$; catenary fillet

- $h^* = 1$
- $h^* = 1.2$
- $h^* = 1.5$
- $h^* = 2$
- $h^* = 3$
- $h^* = 4$
- $h^* = 5$
- $h^* = 6$

$D^* = H^* = 20$
Plane stress
Pure tension
2. Computing the Ideal Catenary Notch

2.4 Results of the catenary analysis

Stress concentration factor $K_t$; catenary fillet

$D^* = H^* = 20$
Plane stress
Pure tension
2. Computing the Ideal Catenary Notch

2.4 Results of the catenary analysis

Von Mises Stress distribution for the different sizes with minimum $K_t$, respectively

- Nominal web stress always 100 MPa, note different legend scaling driven by max. stress!
2. Computing the Ideal Catenary Notch

2.4 Results of the catenary analysis

Von Mises Stress distribution along the outer notch fiber for the different sizes
- Nominal stress always 100 MPa (notch fiber length for web thickness d=10 mm)

- $h^* = 1$
  - $b^* = 0.6217$
  - $t_1 = -1.5238$
  - $K_t = 1.1551$

- $h^* = 1.2$
  - $b^* = 0.6467$
  - $t_1 = -1.7380$
  - $K_t = 1.0855$

- $h^* = 1.5$
  - $b^* = 0.5966$
  - $t_1 = -2.2021$
  - $K_t = 1.0397$

- $h^* = 2$
  - $b^* = 0.5792$
  - $t_1 = -2.684$
  - $K_t = 1.0197$

- $h^* = 3$
  - $b^* = 0.5829$
  - $t_1 = -3.273$
  - $K_t = 1.0076$

- $h^* = 4$
  - $b^* = 0.598$
  - $t_1 = -3.6478$
  - $K_t = 1.0039$

- $h^* = 5$
  - $b^* = 0.6094$
  - $t_1 = -3.9334$
  - $K_t = 1.0023$

- $h^* = 6$
  - $b^* = 0.6177$
  - $t_1 = -4.1654$
  - $K_t = 1.0015$
2. Computing the Ideal Catenary Notch

2.4 Results of the catenary analysis

![Graph showing local stress concentration along the catenary fillet length l_f* for best b* found.](image)

- \( b^* = 0.6467 \)  \( K_t = 1.0855 \)
- \( b^* = 0.5966 \)  \( K_t = 1.0397 \)
- \( b^* = 0.5792 \)  \( K_t = 1.0197 \)
- \( b^* = 0.5829 \)  \( K_t = 1.0076 \)
- \( b^* = 0.6177 \)  \( K_t = 1.0015 \)

D*: \( H^* = 20 \)
Plane stress
Pure tension

**l_f*: outer fiber fillet length normalized by web thickness d
3. Modifying the Catenary Curve

3.1 Idea of the modification

**Result from the previous examination**

- It turns out that with increasing fillet length, the outer fiber notch stress is not well kept close to the nominal stress at many points
- In order to better utilize the material, it may make sense to make the catenary notch more “slim”
- Such a behavior was detected for generalized elliptical fillets (conical rounds) in [2]: By using the parameter “rho”, the ellipse could be scaled and tilted in a suitable way and the stress along the notch outer fiber could be kept closer to the nominal stress
- [3] therefore introduced a third parameter to make the catenary more slim or more fat for a given height h and width b
- On the next slide, this approach is coded in the equation editor
3. Modifying the Catenary Curve

3.2 Coding the modified catenary

**Working with three parameters**
- The modified catenary notch dimensions are preselected by entering h and b.
- If the parameter \( t_1 \) of the modified catenary is similar to the parameter \( t_1 \) of the ideal catenary, belonging to the chosen \( b \), the modified equals the ideal catenary.
- Otherwise, a bigger absolute value of \( t_1 \) makes the notch more slim compared to the ideal catenary and reverse, see [3].
3. Modifying the Catenary Curve

3.3 Analyzing the modified catenary

Sensitivity graph for $h^* = 6$, $b^* = 0.6177$, $t_1 = -1\ldots-6$

- No advantage can be found, ideal catenary is the best!

Notch becomes more fat
(absolute value of $t_1$ decreases)

Notch becomes more slim
(absolute value of $t_1$ increases)

Best point of ideal catenary:
$h^* = 6$
$b^* = 0.6177$
$t_1 = -4.1654$
$K_t = 1.1298$

$K_t = 1.1281$
4. Outlook

4.1 Size comparison of some very good solutions found

Catenary fillet is just surpassed by Baud fillet and conical round (but smaller than the latter)

- Fillet envelopes $h^* \times b^*$
- Shown half model with $d^*/2$
- Other fillets taken from [2]

<table>
<thead>
<tr>
<th>Notch type</th>
<th>No.</th>
<th>$h^*$</th>
<th>$b^*$</th>
<th>$R^*$</th>
<th>$\alpha$ [$^\circ$]</th>
<th>$\rho$</th>
<th>$K_t$</th>
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<td>Conical round</td>
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<td>0.6177</td>
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<td>-</td>
<td>1.0015</td>
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</tbody>
</table>
4. Outlook

4.2 Size comparison of some notches with $K_t \approx 1.05$

Catenary fillet can be made surprisingly small, if small remaining stress concentration is allowed!

- Fillet envelopes $h^* \times b^*$
- Shown half model with $d^*/2$
- Other fillets taken from [2]

<table>
<thead>
<tr>
<th>Notch type</th>
<th>No.</th>
<th>$h^*$</th>
<th>$b^*$</th>
<th>$R^*$</th>
<th>$r^*$</th>
<th>$\alpha$ [°]</th>
<th>rho</th>
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</tbody>
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5. References


Acknowledgement
Thanks to Wigand Rathmann for spending the time to parameterize the catenary curve in a very suitable way for optimization!
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