Active Vibration Control of Axial Piston Machine using Higher Harmonic Least Mean Square Control of Swash Plate

Taeho Kim, Professor Dr.-Ing. Monika Ivantysynova
Maha Fluid Power Research Center, Purdue University, 225 South University Street, West Lafayette, IN 47907-2093, USA, email: kim1489@purdue.edu, mivantys@purdue.edu

Abstract
Noise emission is a major drawback of the positive displacement machine. The noise source can be divided into structure borne noise source (SBNS) and fluid borne noise source (FBNS). Passive techniques such as valve plate optimization have been used for noise reduction of axial piston machines. However, passive techniques are only effective for limited operating conditions or at least need compromises in design. In this paper, active vibration control of swash plate is investigated for vibration and noise reduction over a wide range of operating conditions as an additional method to passive noise reduction techniques. A 75cc pump has been modified for implementation of active vibration control using the swash plate. One tri-axial acceleration sensor and one angle sensor are installed on the swash plate and a high speed servovalve is used for the swash plate actuation. The multi-frequency two-weight least mean square (LMS) filter synthesizes the servovalve input signal to generate a destructive interference force which minimizes the swash plate vibration. An experimental test setup has been realized using Labview field-programmable gate array (FPGA) via cRIO. Simulation and experimental studies are conducted to investigate the possibility of active vibration control.

KEYWORDS: Active vibration control; Axial piston machine; Multi-frequency two-weight LMS filter; Structure Borne Noise Source reduction;

1. Introduction
Positive displacement machines are widely used in many applications of the industry due to their advantages of high power density, controllability, and high efficiency. The noise emission is a major drawback of positive displacement machines. Moreover, it is as an obstacle in widening the application of positive displacement machines. Many researchers have investigated noise reduction of positive displacement machines with different approaches. The main approaches can be categorized into research focused on the transmission path of noise /1/ and research focused on sources of noise /2, 3, 4/.
The noise source can be divided into structure borne noise source (SBNS) and fluid borne noise source (FBNS). Passive techniques such as valve plate optimization have been investigated for noise reduction of axial piston machines /4/. However, passive techniques are only effective for limited operating conditions or at least need compromises in design. Active techniques have been investigated for FBNS reduction /5, 6, 7/. However, few researchers have investigated active techniques for SBNS using swash plate vibration control. A group of researchers in Japan investigated vibration control of the swash plate to reduce SBNS of an axial piston pump by manually tuning the phase and amplitude of synchronized sinusoidal input signals /8, 9/. However, manual tuning leads to limited vibration and noise reduction performance.

In this paper, active vibration control of the swash plate is investigated for vibration and noise reduction over a wide range of operating conditions as an additional method to passive noise reduction techniques. The swash plate moment acts as a disturbance to the pump control system. The swash plate acceleration is a periodic signal, which is composed of harmonic components of fundamental frequencies of the swash plate moment. The concept of active vibration control is to cancel vibration of the swash plate by applying destructive interference signal to the swash plate control. The least mean square algorithm uses the synchronized reference signal for synthesizing destructive interference input, which minimizes swash plate acceleration.

2. Pump modification and swash plate acceleration measurement

A 75cc axial piston pump is modified to have a swash plate acceleration sensor, a swash plate angle sensor, a high speed direct drive servovalve, and a rotation speed sensor for vibration reduction via swash plate control.

![Figure 1: Cutaway drawing of the modified pump](image)
Figure 1 (left) shows the cutaway drawing of the modified pump which has a tri-axial acceleration sensor (PCB W356A12: ±500m/s²), a noncontact type angle sensor (Contelec Vert-X 31E) and a high speed direct drive servovalve (Parker D1FP: 350Hz @5% input, 40LPM). The side cover is redesigned to have an angle sensor and cable path for the sensors. Figure 1 (right) shows an encoder (Heidenhain ROD426) connected to the pump shaft through the auxiliary mount to measure rotation speed. Figure 2 shows the circuit of the dedicated test rig for active vibration control. One hydraulic motor (130cc) is connected with the hydraulic pump (75cc) via shaft coupling in order to drive the pump.

A relief valve is used on the pump delivery line to apply loads to the hydraulic pump. Two flowmeters are installed on the pump delivery line and case drain line to measure the delivery flow rate and case drain flow rate. An oil filter is placed on the external pump control pressure line in order to protect the direct drive servovalve. An accumulator is installed between the oil filter and the direct drive servovalve to minimize pressure drop across the oil filter under fast dynamic conditions. The control and data acquisition of the test rig are implemented using LabVIEW FPGA via NI cRIO-9033. Different actuators and sensors have different control loop frequencies. For example, the active vibration control of the pump, the speed control of the motor, and the pressure sensors / flowmeters used 10KHz, 100Hz, and 100Hz control loop frequencies respectively.

Figure 2: Hydraulic circuit of active vibration control test rig

The acceleration of swash plate is measured using an acceleration sensor mounted 35mm below the rotation axis of the swash plate as shown in Figure 3 (left). The \(a_x\) direction of the acceleration sensor is positioned perpendicular to the radial direction to capture swash plate moment \( (M_x) \). Figure 3 (right) shows the acceleration measurement
in the time domain and frequency domain. The acceleration is measured at the rotation speed of 803rpm, swash plate angle of 6.8deg, delivery pressure of 70bar, and low pressure of 28bar. The acceleration signal is high-pass filtered to remove the DC offset and low-pass filtered using an 800Hz cut-off frequency. The FFT of the acceleration signal shows fundamental frequency and its harmonic components. The fundamental frequency of the swash plate moment ($M_X$) can be calculated from the rotation speed and number of pistons according to Equation (1). In case of an odd number of pistons pump, the principle frequency of $M_X$ is twice the fundamental frequency since there are two peaks per period /10/. The FFT of the swash plate acceleration shows a high peak at the second harmonic frequency as shown in Figure 3 (right).

$$f_{fund} = (\text{rpm}/60) \times z = (803/60) \times 9 = 120.45 \text{ (Hz)}, \quad z: \text{number of piston}$$  \hspace{1cm} (1)

**Figure 3**: Location of swash plate acceleration sensor (left) and swash plate acceleration measurement (right)

### 3. Pump control system modelling

The pump control system consists of the electro-hydraulic module (direct drive servovalve, swash plate angle sensor) and the mechanical module (swash plate, control cylinder, control actuator and linkage). The direct drive servovalve regulates flow rate into the swash plate control cylinder consequently causing linear motion of the control actuator. The linkage system transforms linear motion of the control actuator into rotational motion of the swash plate.

The direct drive servovalve can be modeled using a second order transfer function of the input current and the output stroke of servovalve as equation (2).

$$Y_v(s)/U_v(s) = \frac{\omega_p^2}{s^2 + 2\zeta \omega_p s + \omega_p^2}$$  \hspace{1cm} (2)

The swash plate, the control actuator, and the linkage can be simplified as a single equivalent mass as shown in Figure 4. The equation of motion can be obtained
considering the pressure force \((F_{\Delta p})\), the friction force \((F_F)\) and the self-adjusting force \((F_{SL})\) from swash plate moment \((M_x)\) acting on control actuator, see equation (3).

![Diagram of pump control system](image)

**Figure 4:** Modelling of pump control system

The pressure force \((F_{\Delta p})\) acting on the control actuator can be derived using the pressure difference between the two control cylinders, see equation (5). The friction force \((F_F)\) acting on the control actuator can be modeled using the Stribeck curve, see equation (6). The self-adjusting force \((F_{SL})\) can be calculated from the swash plate moment \((M_x)\) and the lever arm length \((r_{ac})\) as equation (7). The differential pressure \((\Delta p)\) can be calculated using the pressure build-up equation considering flows coming in and out of the control cylinder as equation (8). The common hydraulic capacitance \((C_H)\) is calculated as equation (8). The flow rate from the direct drive servovalve can be calculated from the nominal flow rate of the servovalve as equation (9).

\[
\begin{align*}
F_{\Delta p} &= F_{ac}(p_A - p_B) = A_{ac}\Delta p \\
F_F &= f_c \cdot \text{sign}(\dot{x}) + f_s \cdot \ddot{x} + f_s \cdot e^{-j\pi} \cdot \text{sign}(\dot{x}) \\
F_{SL} &= M_x / r_{ac} \\
\Delta p &= \left( -A_{ac} \dot{x} - \frac{k_{ac}}{2} \Delta p + Q_v \right) \left/ C_H \right. , \quad C_H = \frac{1}{K_{oil}} \left( \frac{1}{V_A} + \frac{1}{V_B} \right)^{-1} = \frac{V_l}{4K_{oil}} \\
Q_v &= Q_{nom} \left( \frac{y_v}{y_{max}} \right) \sqrt{(p_i - \text{sgn}(y_v)\Delta p) / p_{nom}}
\end{align*}
\]

Combining and linearizing equation (2) to (9), the dynamic system of the pump control system can be expressed as equation (10).
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{m_{eq}} (A_{eq} x_3 - f_s x_2) \\
\dot{x}_3 &= \frac{4K_{ev}}{V_t} (-A_{eq} x_2 - \frac{k_2}{2} x_3 + \frac{Q_{nom}}{y_{max}} \sqrt{\frac{P_s}{P_{nom}}} x_4) \\
\dot{x}_4 &= x_5 \\
\dot{x}_5 &= -2\zeta \omega \dot{x}_5 - \omega^2 x_5 + \omega^2 u
\end{align*}
\]

where, \(X = [x_1, x_2, x_3, x_4, x_5]^T = [\dot{x}, \dot{\dot{\dot{p}}}, y, \dot{y}]^T\)

### 4. Model validation

Simulation results are compared with swash plate acceleration measurements to validate the model. Two different sine signals (frequency: 150Hz and 300Hz, amplitude: ±3V (±30%)) are applied to the pump control system at a servovalve supply pressure of 41bar. The direct drive servovalve has a bandwidth of 350Hz at ±5% input signal. The stroke measurements of the direct drive servovalve show that a constant second order transfer function cannot closely estimate the servovalve response at a different input of frequencies. Two different second order transfer functions are used for two different input signals. A signal delay of 700μsec is added to the model acceleration based on the step response test. The comparison between the linear pump model and the real pump using different input signals verified a close match of the hydraulic model.

**Figure 5:** Comparison of pump response (model vs real pump) at different input

### 5. Adaptive LMS filter

Adaptive LMS filters are widely used in the area of active noise control and active vibration control due to their simplicity and stable operation. The error signal \(e(n)\) of an adaptive finite impulse response (FIR) filter can be expressed as equation (11) where, \(d(n)\) is the desired response, \(y(n)\) is the filter output, \(X(n)\) is an input vector, and \(W^T\) is a weight vector of the adaptive FIR filter. The Mean Square Error (MSE) is defined as equation (12). The optimum filter weight vector \(W^*\) can be calculated when the gradient
of the MSE is equal to zero as in equation (13). Combining equations (12) and (13) can be expressed as equation (14) /11, 12/.

\[
e(n) = d(n) - y(n) = d(n) - W^T(n)x(n) \tag{11}
\]

\[
\xi(n) = \mathbb{E}[e^2(n)] = \mathbb{E}[d^2(n)] - 2\mathbb{E}[d(n)x(n)]^T W(n) + W^T(n)\mathbb{E}[x(n)x^T(n)]W(n)
\]

\[
= \mathbb{E}[d^2(n)] - 2P^T W(n) + W^T(n)RW(n) \tag{12}
\]

\[
RW^* = P \tag{13}
\]

The steepest descent algorithm is implemented according to equation (15). The LMS algorithm is an implementation of the steepest descent algorithm with a squared error as an estimation of MSE, see equation (17) /12/.

\[
\xi(n) = \xi_{\min} + [W(n) - W^*]^T R [W(n) - W^*] = \xi_{\min} + V^T(n)RV(n) \tag{14}
\]

\[
W(n+1) = W(n) - (\mu/2)\nabla \xi(n) = W(n) + \mu [P - RW(n)] \tag{15}
\]

\[
\nabla \xi(n) = 2e(n)\nabla e(n) = -2X(n)e(n) \tag{16}
\]

\[
W(n+1) = W(n) - (\mu/2)\nabla \xi(n) = W(n) + \mu X(n)e(n) \tag{17}
\]

6. Active vibration control using higher harmonic two-weight LMS filter with delay compensation

Figure 6 (left) shows the block diagram of a single frequency two-weight LMS filter. When a sinusoidal reference input is applied, the adaptive LMS filter, \( G(z) \) can be modeled as an equivalent linear transfer function as equation (18) /13/, where, \( \mu \) is the convergence factor and \( A \) is the amplitude of reference input.

\[
H(z) = \frac{E(z)}{D(z)} = G(z) \tag{18}
\]

\[
H(z) = \frac{E(z)}{D(z)} = G(z) \tag{19}
\]

\[
H(z) = \frac{E(z)}{D(z)} = G(z) \tag{20}
\]

Figure 6: Block diagram of two-weight LMS notch filter

The generalized closed loop transfer function (\( H(z) = E(z)/D(z) \)) for length \( L \) was derived as equation (19) /13/. The poles of \( H(z) \) can be calculated as equation (20). The stability of \( H(z) \) changes according to the value of convergence factor \( \mu \). As \( \mu \) increases, the
poles increase and exits the unit circle when \( \mu L A^2 / 4 > 1 \). Therefore the stability condition of \( H(z) \) can be written as equation (21).

\[
G(z) = Y(z) / E(z) \approx \mu A^2 \left[ (z \cos \omega - 1) / (z^2 - 2(z \cos \omega + 1) \right] \\
H(z) = \left[ (z^2 - 2 z \cos \omega + 1) / (z^2 - (2 - \mu L A^2 / 2) z \cos \omega + 1 - \mu L A^2 / 2) \right]
\]

\[
z_p = r_p \exp(\pm j \theta_p), \quad r_p = \sqrt{1 - \mu L A^2 / 2}, \quad \theta_p = \cos^{-1} \left[ (1 - \mu L A^2 / 4) \cos \omega / \sqrt{1 - \mu L A^2 / 2} \right]
\]

\[
0 < \mu < 4 / LA^2
\]

Delay compensation can be used to compensate the phase change of the pump control system as shown in Figure 6 (right). The output of a single frequency two-weight LMS filter with delay compensation can be calculated as equation (22). The weights are updated via LMS algorithm according to equation (23).

\[
y(n) = w_0(n) x_0(n) + w_1(n) x_1(n), \quad x_0(n) = A \cos(w_0 n), \quad x_1(n) = A \sin(w_0 n)
\]

\[
w_0(n+1) = w_0(n) + \mu x_0(n-\Delta) e(n), \quad w_1(n+1) = w_1(n) + \mu x_1(n-\Delta) e(n)
\]

A multi-frequency two-weight LMS filter with delay compensation is applied to cancel multiple harmonic components of swash plate vibration as shown in Figure 7.

**Figure 7:** Block diagram of higher harmonic LMS active vibration control

The multi-frequency two-weight LMS filter is formulated by connecting single frequency two-weight LMS filters of different harmonic frequencies in parallel. Thus, individual adjustment of the convergence factor (\( \mu \)) and delay (\( \Delta \)) is possible at each frequency. The reference signals are synchronized with the rotational speed of the pump to precisely estimate fundamental frequency of swash plate acceleration.
7. Active vibration control simulation

The multi-frequency two-weight LMS filter is simulated with the linear pump model in MatLab. The acceleration signal used in the simulation is recreated from the measured acceleration signal using least square fitted Fourier series with 16 harmonic components, see Figure 8. The acceleration signal is added to the output of pump model as a disturbance. The frequency of the reference signal is set to the fundamental frequency of the swash plate acceleration assuming the synchronized reference signal. Signal delay is added between the active vibration controller and the pump model to reflect delays introduced by the sensors and electronics. The second order transfer function of bandwidth 350Hz is used for the direct drive servovalve model.

![Figure 8: Recreated swash plate acceleration using Fourier series](image)

**Figure 8:** Recreated swash plate acceleration using Fourier series

**Figure 9** (left) shows the swash plate acceleration from dynamic simulation of the multi-frequency two-weight LMS filter. The vibration control is turned on from 10sec to 40sec. The active vibration control reduced the swash plate acceleration approximately 30%. The active vibration controller utilized 5 harmonic frequencies from fundamental to fifth harmonic frequency.

![Figure 9: Acceleration in simulation (time domain vs frequency domain)](image)
**Figure 9** (right) shows a comparison of acceleration FFTs between ‘with AVC’ and ‘without AVC’ to see the acceleration reduction at targeted harmonic frequencies (fundamental to fifth harmonics). The reduction of the first 5 harmonics shows the influence of active vibration control. The FFT shows that the residual acceleration with vibration control consists of higher harmonic components. **Figure 11** (above) shows 10 weights of 5 two-weight LMS filters which converge to stable values while the active vibration control is turned on.

8. Active vibration control measurement

**Figure 10** (left) shows the swash plate acceleration from the real pump measurement of multi-frequency two-weight LMS filter. The fundamental frequency is generated from the rotational speed of the pump and feed into the reference signal generator of FPGA. The swash plate vibration control is turned on from 2 sec to 37 sec. The active vibration control reduced the swash plate acceleration approximately 30%. In the real pump test, different sets of convergence factors and delays are used at each harmonic frequency. The single convergence factor and delay for all harmonic frequencies could not provide stable active vibration control results.

**Figure 10**: Acceleration in measurement (time domain vs frequency domain)

**Figure 10** (right) compares FFTs of the measured acceleration between ‘with AVC’ and ‘without AVC’. The comparison shows an acceleration reduction at the first 5 harmonic components with active vibration control. At the same time, a slight increase of higher harmonic components is observed from the FFT. **Figure 11** shows weights of the adaptive LMS filters from the simulation (top) and the measurement (bottom). All weights converged to stable values with active vibration control. Weights of the second harmonic are the largest values showing that most of the input signals are in low frequency. Individual tuning of convergence factors and delays at each harmonic frequency resulted in a stable vibration reduction under steady state conditions. Different values of delays and convergence factors were needed for some different operation conditions, since the
characteristics of direct drive servovalve changes depending on the frequency and amplitude of input signal, which are dependent on the operating conditions.

Figure 11: Comparison of weights (simulation vs measurement)

9. Conclusion
The aim of this paper is to study the possibility of active vibration reduction using direct swash plate control for structure borne noise source (SBNS) reduction. The active vibration control can be used as a complementary noise reduction method for existing passive noise reduction techniques to overcome their limitations. A 75cc axial piston pump was modified for active vibration control of the swash plate including a tri-axial acceleration sensor, a swash plate angle sensor, a rotation speed sensor, and a high speed servovalve. A new test rig is created for active vibration control research allowing acceleration measurement and high speed active vibration control for a wide range of operating conditions. Measurement and simulation results showed good acceleration reduction performance using multi-frequency two-weight LMS filter with delay compensation under steady state conditions. The measurement results also showed a slight increase of higher harmonic components with active vibration control. The result of this investigation demonstrated the possibility of SBNS reduction via direct swash plate control.

10. References


11. Nomenclature

\[ M \] Swash plate moment \hspace{1cm} Nm

\[ a \] Acceleration \hspace{1cm} m/s^2

\[ f_{\text{fund}} \] Fundamental frequency of swash plate moment \hspace{1cm} Hz

\[ Z \] Number of pistons \hspace{1cm} -

\[ r_{\text{sensor}} \] Sensor position from the rotational axis \hspace{1cm} m

\[ r_{ac} \] Swash plate lever arm length \hspace{1cm} m

\[ F \] Force \hspace{1cm} N

\[ p \] Pressure \hspace{1cm} bar

\[ Q \] Flow rate \hspace{1cm} m^3/s

\[ A_{ac} \] Control actuator piston area \hspace{1cm} m^2

\[ x \] Control actuator displacement \hspace{1cm} m

\[ m \] Mass \hspace{1cm} kg

\[ C_H \] Hydraulic capacitance \hspace{1cm} m^5/N

\[ \Theta_{\text{AS}} \] Effective inertia of adjustment system \hspace{1cm} kg\cdot m^2

\[ f_c \] Coefficient of Coulomb friction force \hspace{1cm} N

\[ f_v \] Coefficient of viscous friction \hspace{1cm} N\cdot s/m

\[ f_s \] Coefficient of static friction force \hspace{1cm} N

\[ K_{\text{oil}} \] Bulk modulus of oil \hspace{1cm} Pa

\[ k_{li} \] Leakage coefficient from control cylinder to case \hspace{1cm} m^3/Pa\cdot s

\[ V \] Volume \hspace{1cm} m^3

\[ y_V \] Servovalve stroke \hspace{1cm} m

\[ \omega_V \] Servovalve natural frequency \hspace{1cm} rad/s
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta_v$</td>
<td>Servovalve damping ratio</td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>Error of adaptive filter</td>
<td></td>
</tr>
<tr>
<td>$d'$</td>
<td>Desired response of adaptive filter</td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td>Output of adaptive filter</td>
<td></td>
</tr>
<tr>
<td>$W$</td>
<td>Weight vector of adaptive filter</td>
<td></td>
</tr>
<tr>
<td>$W^*$</td>
<td>Optimum weight vector</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>Convergence rate of adaptive filter</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>Length of adaptive filter</td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frequency of reference signal</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Delay in signal</td>
<td>$\mu$sec</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Mean Square Error</td>
<td></td>
</tr>
<tr>
<td>$\nabla$</td>
<td>Gradient operator</td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>Input cross-correlation matrix</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>Input correlation matrix</td>
<td></td>
</tr>
</tbody>
</table>