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Trade, Inequality, and the Size of the Welfare State

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Abstract
This paper investigates the effects of international trade in a general equilibrium model with heterogeneous firms where a welfare state redistributes income. We look at a very stylised progressive non-distortionary redistribution scheme. We show that for a given tax rate international trade increases income per capita, but also leads to higher income inequality. Two aspects of income inequality are examined. First, inter-group inequality between managers and workers is considered. Second, intra-group inequality within the group of managers is investigated. For a given tax rate the size of the welfare state and therefore the transfer per capita increases when going from autarky to trade. This second-round effect counteracts the primary increase in inequality, yet cannot outweigh it. Since the redistribution scheme is non-distortionary, it is possible to decrease trade-induced inequality by increasing the tax rate without jeopardising the gains from trade.

JEL classification: D31; F12; F16; H24; H25

Keywords: International trade; Income inequality; Redistribution; Heterogeneous firms

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1 Introduction

Distributional effects of globalisation are a crucial concern in the public debate. It is also a classical question in academia. Research on this topic dates back to the famous work by Stolper and Samuelson (1941). In the last few years we saw a growing body of literature that addresses the link between international trade and the income distribution exploiting heterogeneity 1. What is generally found is that there are gains from trade but these overall gains come at a cost: a rise in income inequality. This trade-induced rising inequality might lead to a protectionist drift in society as argued by Scheve and Slaughter (2007). Therefore it might be important to accompany trade liberalisation with carefully chosen redistribution policies in order for the current levels of economic integration to be sustainable. According to Scheve and Slaughter (2007) “a New Deal for globalisation” is needed - linking trade liberalisation to redistribution policies. This paper sheds light on exactly this issue. It is therefore vital to explicitly model a welfare state and then look at the picture of trade-induced inequality. We want to know how trade affects inequality in the presence of a welfare state and whether it is possible to decrease trade-induced inequality without jeopardising the gains from trade. Therefore this paper integrates a welfare state into a new trade theory model that links globalisation and inequality.

In terms of the basic model setup this paper is closely linked to Egger and Kreickemeier (2012), which is a trade model of monopolistic competition with heterogeneous firms. Actually we use a simplified version of this model with perfect labour markets. The basic set-up is as follows. Individuals are heterogeneous in their managerial skill. According to their skill individuals choose whether to become a manager or a worker. Individuals can use their managerial skill only if they decide to become managers. If they become workers, they can not use their managerial skill. Hence, managers are heterogeneous and it is assumed that the ability of the manager determines the productivity of the firm they are running. Managerial income is given by the profits of the firm. Workers, in contrast, are paid the same wage regardless in which firm they are employed. In each firm there will be one manager and an endogenous number of workers. The welfare state is modelled as follows. We look at a non-distortionary tax-transfer system. The redistribution scheme is financed by a proportional tax on both profit and wage income and gives the same absolute transfer to all individuals.2 The occupational choice is modelled as in Lucas (1978). Individuals compare their income in the two occupations. Hence the labour indifference condition implicitly determines the ability of the marginal manager. The labour indifference condition together with the resource constraint pins down the equilibrium factor allocation in the economy. Welfare is calculated in a

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1 See Egger and Kreickemeier (2009a, 2012); Helpman et al. (2010); Yeaple (2005). For a non-technical overview on this topic see Harrison et al. (2011).

2 This redistribution scheme can be seen as a very stylised representation of a progressive tax-transfer system.
utilitarian way as per capita income. The model features both inter-group inequality, i.e. inequality between the group of managers and the group of workers, and intra-group inequality, i.e. inequality within the group of managers. Inter-group inequality is calculated as the ratio between average post tax-transfer profit income and post tax-transfer labour income. Intra-group inequality is determined using the Gini criterion. Trade is considered between two symmetric countries, Home and Foreign. There are two types of costs involved with trade. First, there is the standard iceberg transport cost. Second, there is a fixed cost involved with exporting, since an export consultant needs to be hired. This export consultant is paid the economy wide wage. This leads to self-selection of the most productive firms into exporting.

As shown in Egger and Kreickemeier (2012) trade leads to aggregate welfare gains. The reason for this finding is the higher cutoff productivity in the open economy that leads to an efficiency gain for the economy. However, ceteris paribus, the open economy equilibrium features also higher inequality: both higher inter-group inequality and higher intra-group inequality among managers measured by the Gini coefficient. We find that the non-distortionary redistribution scheme considered does not affect the occupational choice of individuals. The implication of this finding is that the cutoff ability level, the factor allocation and welfare do not depend on the tax-transfer system. It is therefore possible to decrease trade-induced inequality by accompanying trade liberalisation with an increase in the tax rate. Furthermore we find that due to gains from trade the size of the welfare state in the open economy is higher than the size of the welfare state in the closed economy. This leads to an increase in the transfer per capita - an effect that c.p. decreases inequality. However, this non-trivial second-round effect cannot outweigh for the primary trade-induced increase in inequality.

This paper is linked to the heterogeneous firms trade literature that was started by Melitz (2003). There are several papers that look at distributional effects of globalisation in models of heterogeneous agents (see Egger and Kreickemeier, 2009a, 2012; Helpman et al., 2010; Yeaple, 2005). Crucial to our paper is an occupational choice mechanism that is modelled as in Lucas (1978). Models in international trade that use occupational choice are Antràs et al. (2006), Monte (2011), Egger and Kreickemeier (2012) and Egger et al. (2015). Since the model features inequality in managerial income, it is also related to the literature that tries to explain CEO payments (see Gabaix and Landier, 2008; Gabaix et al., 2014; Gersbach and Schmutzler, 2007). To the best of our knowledge the functioning of a welfare state in international trade models with heterogeneous firms, which is the focus of our paper, is not thoroughly investigated in the literature so far. There are only a few other papers that look at similar issues, in different frameworks though. Egger and Kreickemeier (2009b) introduce a redistribution scheme in a trade model where a fair-wage effort mechanism leads to firm-specific wages. They show that with a suitably chosen increase in the
profit tax rate higher welfare and a more equal income distribution than in autarky is possible under trade. However with production labour as the only input factor, this model cannot address inter-group inequality and intra-group inequality among the group of managers. Hence, it can be seen complementary to what we are doing. Itskhoki (2008) focuses on the optimal policy response to trade-induced inequality and the trade-off between efficiency and equality. He shows that the optimal policy response to trade-induced inequality may be to decrease marginal tax rates. There is another recent paper by Antràs et al. (2016) that looks at redistributing gains from trade when redistribution is costly. However, the trade model they consider is much more parsimonious than the trade model in our paper. Furthermore, their model is not suitable to discuss inter-group inequality and intra-group inequality among managers.

The remainder of this paper is organised as follows. In Section 2 we derive the closed economy equilibrium, i.e. we solve for the factor allocation, welfare and the income distribution. In Section 3 we look at the open economy equilibrium and discuss the effects of trade. Section 4 concludes by summarizing the most important results and puts the contribution of this very paper into context.

2 The closed economy

Individuals
Following Egger and Kreickemeier (2012) we assume an economy that is populated by individuals where the mass of individuals is denoted by N. These individuals are heterogeneous in their managerial ability. Individuals can use their managerial ability if they decide to become a manager (mass of managers denoted by M). However, they cannot use their managerial ability in the role as workers (mass of workers denoted by L). Managers earn the firm’s profit whereas workers are paid an economy wide wage.

The final goods sector
Final output is a homogeneous good. Following Ethier (1982) and Egger et al. (2015) let final output be a CES-aggregate of all varieties \( v \), such that

\[
Y = \left[ \int_{v \in V} q(v)^{\frac{\sigma - 1}{\sigma}} \, dv \right]^{\frac{\sigma}{\sigma - 1}},
\]

where \( q(v) \) is the quantity of variety \( v \), \( V \) is the set of all available varieties with measure \( M \) and \( \sigma > 1 \) is the elasticity of substitution between the different varieties of intermediate goods. The production function has external increasing returns to scale and is homogeneous of degree \( \sigma/(\sigma - 1) \) with respect to the number of varieties, which is the standard textbook case. Since final output is chosen as the numéraire its price is normalised to one. We assume perfect competition in the final
goods sector and therefore get the following demand function for each variety \( v \)

\[
q(v) = Y p(v)^{-\sigma},
\]

where \( p(v) \) is the price of variety \( v \).

**The intermediate goods sector**

We assume monopolistic competition in the intermediate goods sector. The mass of intermediate goods producers is equal to \( M \). In each firm there will be one manager and an endogenous number of workers, \( l(v) \). Managers are heterogeneous in their ability. We assume that the ability of the manager determines firm productivity, \( \varphi(v) \). The number of workers employed in a firm is proportional to output, i.e. \( q(v) = \varphi(v) l(v) \). The unit production costs for each firm are given by \( c(v) = w/\varphi(v) \), where \( w \) is the economy wide wage. Since individuals cannot use their managerial ability if they decide to become a worker, workers are identical and hence are paid the same wage, \( w \). Profit maximisation at the level of intermediate goods producers gives the price of each variety as a constant mark-up over marginal costs

\[
p(v) = \frac{\sigma}{\sigma - 1} c(v),
\]

where the price is firm specific because marginal costs are firm specific. Equilibrium output \( q(v) \) and equilibrium revenue \( r(v) \) in the intermediate goods sector are given by

\[
q(v) = Y \left( \frac{\sigma}{\sigma - 1} c(v) \right)^{-\sigma} \quad \text{and} \quad r(v) = Y \left( \frac{\sigma}{\sigma - 1} c(v) \right)^{1-\sigma}.
\]

Hence, operating profits are given by

\[
\pi_{op} = \frac{1}{\sigma} r(v).
\]

By calculating relative output, employment, revenues and profits of two firms 1 and 2 we get

\[
\frac{q(v_1)}{q(v_2)} = \left( \frac{\varphi(v_1)}{\varphi(v_2)} \right)^\sigma \quad \text{and} \quad \frac{l(v_1)}{l(v_2)} = \frac{r(v_1)}{r(v_2)} = \frac{\pi_{op}(v_1)}{\pi_{op}(v_2)} = \left( \frac{\varphi(v_1)}{\varphi(v_2)} \right)^{\sigma-1}.
\]

We hence see that firms run by managers with higher managerial ability have higher output, revenues and operating profits and employ more workers.

**Average productivity**

As it is very common in this literature we assume productivity to be Pareto distributed. The distribution function is given by \( G(\varphi) = 1 - \left( \frac{\varphi_{min}}{\varphi} \right)^k \), where the lower bound, \( \varphi_{min} \) is normalised
to one. We can then relate average productivity $\bar{\varphi}$ to the cutoff productivity $\varphi^*$

$$\bar{\varphi} = \left(\frac{k}{k - (\sigma - 1)}\right)^{1/(\sigma - 1)} \varphi^*,$$

(7)

where $k > \sigma - 1$ is assumed.\(^3\)

The redistribution scheme

We now want to introduce a simple redistribution scheme. The redistribution scheme is financed by a proportional tax on income (both profit and wage income) and this tax revenue is then lump-sum redistributed to all individuals. The tax rate is imposed by the government and given by $t \in (0, 1)$. Taking into account the budget constraint of the government, the transfer income $b$ for each individual is equal to

$$b = \frac{tM\bar{\pi} + tLw}{N}.$$

(8)

Occupational choice

According to their skill individuals decide whether to become a manager or a worker (see Lucas, 1978; Egger and Kreickemeier, 2012). They compare their income in the two occupations. Hence, the labour indifference condition that implicitly determines the ability of the marginal manager is given by

$$(1 - t)w + b = (1 - t)\pi(\varphi^*) + b,$$

(9)

where $\varphi^*$ denotes the ability of the marginal manager. This means that individuals with an ability $\varphi \geq \varphi^*$ choose to become managers. Individuals with $\varphi < \varphi^*$ choose to become workers. Having a closer look at Eq. (9) it becomes obvious that the labour indifference condition can easily be reduced to

$$w = \pi^*.$$

(10)

This implies that the occupational choice and hence the ability of the marginal manager is not affected by the tax-transfer system. This finding has important implications for the factor allocation and welfare in the economy as will be discussed in what follows.

Equilibrium factor allocation

Aggregate profits are given by $\Pi = \frac{1}{\sigma}Y$. Aggregate labour income is given by $wL = \frac{\sigma - 1}{\sigma}Y$. The labour indifference condition, Eq. (10), states that the labour income per worker, needs to be equal

\(^3\)The derivation of Eq. (7) can be found in the Appendix.
to the profits of the marginal firm. Using Eq. (6) together with Eq. (7) we get

\[
\frac{(\sigma - 1)Y}{\sigma L} = \frac{Y}{\sigma M} \left( \frac{k - (\sigma - 1)}{k} \right).
\] (11)

We can solve Eq. (11) for the total mass of workers \( L \) and get

\[
L = \frac{k(\sigma - 1)}{k - (\sigma - 1)} M,
\] (12)

which gives an upward sloping relationship between the mass of workers and the mass of managers as illustrated in Figure 1. An increase in the mass of workers reduces the labour income per worker ceteris paribus, therefore the mass of managers also has to increase (thereby reducing the profit of the marginal manager) in order to restore indifference. The mass of individuals is denoted by \( N \) and is a parameter in this model. Since individuals choose to become either managers or worker, the resource constraint of this economy is given by

\[
L = N - M.
\] (13)

The resource constraint is a downward sloping locus in the \( M - L \) space as depicted in Figure 1. The resource constraint together with the labour indifference condition determines the mass of managers and workers in the economy as follows

\[
M_a^n = \frac{k - (\sigma - 1)}{k\sigma - (\sigma - 1)} N \quad \text{and} \quad L_a^n = \frac{k(\sigma - 1)}{k\sigma - (\sigma - 1)} N.
\] (14)

Please note that the subscript \( a \) indicates the autarky equilibrium and the superscript \( n \) shows that we are looking at the non-distortionary tax-transfer system. We see that the equilibrium factor allocation does not depend on the redistribution scheme looked at in this section. The cutoff ability is implicitly determined by \( M = [1 - G(\varphi^*)] N \). Solving explicitly for the cutoff ability we get

\[
\varphi_a^{n*} = \left( \frac{k\sigma - (\sigma - 1)}{k - (\sigma - 1)} \right)^{\frac{1}{\sigma}}.
\] (15)

As depicted in Figure 1 below there is a downward sloping relationship between the mass of managers and the cutoff ability. If all individuals in the economy decide to become managers, the cutoff ability is at its lower bound which is normalised to one. The cutoff ability does not depend on the tax-transfer system.
Welfare

Welfare is calculated in a utilitarian way as per capita income \( Y/N \). Aggregate income is given by

\[
Y = \sigma \Pi = \sigma M \pi(\tilde{\phi}).
\]  

(16)

Profits of the firm with average productivity can be calculated as follows

\[
\pi(\tilde{\phi}) = \frac{k}{k - (\sigma - 1)} w = M^{\frac{1}{\sigma - 1}} \frac{k}{k - (\sigma - 1)} \frac{\sigma - 1}{\sigma} \hat{\phi} = M^{\frac{1}{\sigma - 1}} \left( \frac{k}{k - (\sigma - 1)} \right)^{\frac{\sigma}{\sigma - 1}} \frac{\sigma - 1}{\sigma} \varphi^*. 
\]  

(17)

The first equality sign relates profits of the average firm to profits of the marginal firm and uses the labour indifference condition. The second equality sign uses the mark-up pricing rule for the average firm. Note that the price of the average firm is given by \( p(\tilde{\phi}) = M^{\frac{1}{\sigma - 1}} \). The third equality sign uses the relationship between the marginal and the average productivity. Hence, we can calculate aggregate income as follows

\[
Y^a_n = M^{\frac{1}{\sigma - 1}} \left( \frac{k}{k - (\sigma - 1)} \right)^{\frac{\sigma}{\sigma - 1}} (\sigma - 1) \varphi^*. 
\]  

(18)

Using the equilibrium values for the mass of firms \( M \) and for the cutoff ability \( \varphi^* \) we get the following expression for per capita income

\[
\left( \frac{Y}{N} \right)^a_n = (\sigma - 1) \left( \frac{k}{k\sigma - (\sigma - 1)} \right)^{\frac{\sigma}{\sigma - 1}} \left( \frac{k\sigma - (\sigma - 1)}{k - (\sigma - 1)} \right)^{\frac{\sigma}{\sigma - 1}} N^{\frac{1}{\sigma - 1}}. 
\]  

(19)

We see from this expression that welfare depends on the mass of individuals in the sense that larger
economies (higher \( N \)) enjoy higher welfare. Furthermore we see that welfare does not depend on the redistribution scheme. The tax-transfer system considered in this section is non-distortionary.

**Size of the welfare state**

We next want to determine the size of the welfare state. Looking at the budget constraint of the government, the size of the welfare state is given by \( \text{SizeWelfareState}_n = t [M \bar{\pi} + Lw] \). Expressing all endogenous variables in terms of exogenous parameters, the size of the welfare state is given by

\[
\text{SizeWelfareState}_n = t \left[ (\sigma - 1) \left( \frac{k}{k\sigma - (\sigma - 1)} N \right)^{\frac{\sigma}{\sigma - 1}} \left( \frac{k\sigma - (\sigma - 1)}{k - (\sigma - 1)} \right)^{\frac{1}{\sigma}} \right]. \tag{20}
\]

**Income distribution**

We next want to look at the income distribution in the economy. The first inequality measure we consider is the inter-group inequality, i.e. inequality between the group of managers and the group of workers. Inter-group inequality is calculated as the ratio between average post tax-transfer managerial income and post tax-transfer labour income. Hence it is given by the following ratio

\[
\text{InterIneq}_n = \frac{(1 - t)\bar{\pi} + b}{(1 - t)w + b}, \tag{21}
\]

where the numerator denotes after tax profit income plus transfer income and the denominator denotes after tax wage income plus transfer income. Noting that by virtue of the government budget constraint the transfer \( b \) is equal to \( \frac{tM\bar{\pi} + tLw}{N} \), taking into account that \( \bar{\pi} = \pi(\bar{\varphi}) = \frac{k}{k - (\sigma - 1)} w \) and using the equilibrium factor allocation, we can calculate inter-group inequality as

\[
\text{InterIneq}_n = \frac{(1 - t)k}{k - (\sigma - 1)} + \frac{t k\sigma}{k\sigma - (\sigma - 1)} \frac{1}{1 - t + \frac{t k\sigma}{k\sigma - (\sigma - 1)}}. \tag{22}
\]

For \( t = 0 \) this expression collapses to \( \frac{k}{k - (\sigma - 1)} \) which is greater than unity and represents inter-group inequality in an economy without a welfare state redistributing income. Clearly, for \( t = 1 \) inter-group inequality is equal to unity, i.e. there is no inequality between managers and workers anymore. It is straightforward to show that \( \frac{\partial \text{InterIneq}_n}{\partial t} < 0 \). This means that inter-group inequality decreases in the tax rate chosen.

The second inequality measure we consider is intra-group inequality within the group of managers. Intra-group inequality is determined using the Gini criterion, that can be derived from the Lorenz curve. As a first step we therefore calculate the cumulative managerial income of all managers with a productivity level lower than or equal to \( \bar{\varphi} \in [\varphi^*, \infty] \) relative to the aggregate managerial income. With the redistribution scheme in place managerial income consists of profit
income after taxation plus transfer income. Hence, we get the following expression

\[
(1 - t)\Pi(\bar{\varphi}) + bM(\bar{\varphi})
\]

\[
(1 - t)\Pi + bM.
\]

The share of all firms with a productivity smaller than or equal to \(\bar{\varphi}\) is given by

\[
\gamma = 1 - \left(\frac{\varphi^*}{\bar{\varphi}}\right)^k.
\]

We can get the following Lorenz curve for managerial income after redistribution\(^4\)

\[
Q^n_a(\gamma) = \frac{(1 - t)k^{-k(\sigma - 1)}k(1 - \gamma)^{k - (\sigma - 1)}k - (1 - t)k^{-k(\sigma - 1)}k + t}\frac{k\sigma - (\sigma - 1)}{k\sigma - (\sigma - 1)}(1 - \gamma)}{(1 - t)k^{-k(\sigma - 1)}k + t}\frac{k\sigma - (\sigma - 1)}{k\sigma - (\sigma - 1)}.
\]

The Lorenz curve for managerial income shows the share of managerial income that goes to the lowest \(\gamma \times 100\) percent of firms in the managerial income distribution. The Gini coefficient can then be computed as

\[
\text{IntraIneq}_a^n = 1 - 2\int_0^1 Q^n_a(\gamma)d\gamma = \frac{(1 - t)k^{-k(\sigma - 1)}2k^{-k(\sigma - 1)} - t}\frac{k\sigma - (\sigma - 1)}{k\sigma - (\sigma - 1)}.
\]

For \(t = 0\) the Gini coefficient collapses to \(\frac{\sigma - 1}{2k - (\sigma - 1)}\) which represents the Gini coefficient in an economy with no redistribution scheme in place. For \(t = 1\) the Gini coefficient is equal to zero, which means that there is perfect equality within the group of managers. It is straightforward to show that \(\frac{\partial \text{IntraIneq}_a^n}{\partial t} < 0\). This means that intra-group inequality is decreasing in the tax rate.

3 The open economy

Trade costs

In this section the trading equilibrium will be described. Trade takes place between two identical countries, Home and Foreign. There are no asymmetries between the two countries. In particular this implies that also the policy dimension, i.e. the tax rate, is the same in both countries. Trade is modeled along the lines of Krugman (1980), Ethier (1982) and Melitz (2003). As in Egger and Kreickemeier (2012), there are two types of costs involved with trade. First, there is the standard iceberg transport cost, \(\tau > 1\). Second, there is a fixed cost involved with exporting since an export consultant needs to be hired. The export consultant is paid a fee \(f\), which is also endogenous and equal to the economy wide wage.

\(^4\)The derivation of the Lorenz curve can be found in the Appendix.
Redistribution scheme

In this section the effects of trade when a non-distortionary redistribution scheme is in place will be described. As in the closed economy the redistribution scheme is financed by a proportional tax on income and this tax revenue is then lump-sum redistributed. Note that in the open economy there are three possible occupations generating income. First, there is profit income of managers. Second, there is wage income of workers. Third, there is the income of export consultants. Taking into account the budget constraint of the government the transfer income for each individual is equal to

\[ b = \frac{tM\bar{\pi}_o + tLw + t\chi Mf}{N}. \]  

(27)

Please note that \( \bar{\pi}_o \) denotes average profits in the open economy and \( \chi \) denotes the share of exporting firms in the economy.

Occupational choice

We now want to analyse how the occupational choice by individuals changes in the open economy. In the open economy individuals can choose among three occupations. Either they become a manager or a worker or an export consultant. Therefore the indifference condition needs to be modified as follows

\[ (1 - t)\pi(\varphi^*) + b = (1 - t)w + b = (1 - t)f + b. \]  

(28)

The post tax-transfer profits of the marginal firm need to be equal both to the post tax-transfer labour income and to the post tax-transfer export consultant income. We see that it can easily be reduced to

\[ \pi(\varphi^*) = w = f. \]  

(29)

We see that Eq. (29) does not depend on the redistribution scheme. Hence, we can conclude that also in the open economy the tax-transfer system does not affect the occupational choice of individuals.

The decision to export

Overall operating profits of an exporting firm with domestic revenues \( r(\varphi) \) are equal to \( \sigma^{-1}\Omega r(\varphi) \), with \( 1 < \Omega \equiv 1 + \tau^{1-\sigma} \leq 2 \). We can formulate the indifference condition for the marginal exporter as follows

\[ (1 - t)\left( \frac{\Omega r(\varphi^*)}{\sigma} - f \right) + b = (1 - t)\frac{r(\varphi^*)}{\sigma} + b. \]  

(30)

This means that managers with an ability \( \varphi \geq \varphi^*_\chi \) choose to become exporters, whereas managers
with an ability \( \varphi < \varphi^* \) choose to stay non-exporters. Remember that the fixed cost is endogenous. In equilibrium the income of the export consultant will just be equal to the profit of the marginal firm. Hence, using the fact that \( f = \pi(\varphi^*) = \frac{r(\varphi^*)}{\sigma} \) we can rewrite the exporting indifference condition as

\[
\Omega^n = 1 + \left( \frac{\varphi^*}{\varphi_X} \right)^{\sigma-1}.
\] (31)

The share of exporting firms is denoted by \( \chi \) and can be calculated as

\[
\chi^n = \frac{1 - G(\varphi^*)}{1 - G(\varphi_X)} = \left( \frac{\varphi^*}{\varphi_X} \right)^k = (\Omega - 1)^{\frac{k}{\sigma-1}} = \tau^{-k}.
\] (32)

We see that there is a direct link between the iceberg transport cost \( \tau \) and the share of exporting firms \( \chi \), with \( 0 < \chi < 1 \). Clearly, the share of exporting firms is declining in the amount of iceberg transport cost. Also worth mentioning is the fact that the share of exporting firms does not depend on the tax-transfer system.

**Average productivity**

As done for the closed economy we define \( \tilde{\varphi} \) as the weighted productivity average of all firms selling in this particular market. We can then write the aggregates in the open economy as follows

\[
Y = (M(1 + \chi))^{\frac{1}{\sigma-1}} q(\tilde{\varphi}), \quad R = M(1 + \chi) r(\tilde{\varphi}) \quad \text{and} \quad \Pi = M(1 + \chi) \pi(\tilde{\varphi}).
\] (33)

**Equilibrium factor allocation**

In this part the equilibrium factor allocation in the open economy is described and compared to the autarky equilibrium. Starting point for this analysis is the labour indifference condition for the open economy, i.e. \( \pi(\varphi^*) = w = f \). Using the relationships that \( \pi(\tilde{\varphi}) = r(\tilde{\varphi})/\sigma, \quad \pi(\tilde{\varphi})/\pi(\varphi^*) = k/(k - (\sigma - 1)), \quad (\sigma - 1)/\sigma Y = Lw \) and \( Y = M(1 + \chi) r(\tilde{\varphi}) \) the labour indifference condition can be written as

\[
L = \frac{k(\sigma - 1)(1 + \chi^n)}{k - (\sigma - 1)} M.
\] (34)

This is one of the two important expressions in determining the equilibrium factor allocation. It gives a positive relationship between the mass of firms and the mass of workers in the economy. The labour indifference condition is illustrated in Figure 2. We see that it is different from the labour indifference condition under autarky that has been derived in Section 2. We find that in the open economy the labour indifference condition is less steep in the \( L - M \) space. As outlined in Egger and Kreickemeier (2012) the argument runs as follows. A given level of operating profits in the open economy is associated with lower profits of the marginal firm since part of the profits
are earned through exporting activity. Therefore the wage also has to fall in order to restore indifference which is achieved via an increase in the mass of workers. The second relationship that we need to consider is the resource constraint. It is also different from the resource constraint under autarky. In the open economy the resource constraint is given by

\[ N = L + M + \chi^n M. \]  

(35)

It shows that individuals in the open economy are either employed as production workers or run a firm as a manager or work as export consultants for an exporting firm. The resource constraint is also illustrated in Figure 2. Together the labour indifference condition and the resource constraint determine the equilibrium mass of firms, \( M \) and the mass of workers, \( L \)

\[ M_o^n = \frac{k - (\sigma - 1)}{(1 + \chi^n)(k\sigma - (\sigma - 1))} N \quad \text{and} \quad L_o^n = \frac{k(\sigma - 1)}{k\sigma - (\sigma - 1)} N. \]  

(36)

Comparing the equilibrium factor allocation under trade with the equilibrium factor allocation under autarky we find that the mass of workers stays the same, but the mass of firms is smaller in the open economy compared to the autarky equilibrium. It is now straightforward to derive the cutoff ability in the open economy. The cutoff ability is implicitly given by the relation \( M = [1 - G(\varphi^*)]N \). Solving for the cutoff ability we get

\[ \varphi_o^{n*} = \left( \frac{N}{M} \right)^{\frac{1}{\tau}} = \left( \frac{(1 + \chi^n)(k\sigma - (\sigma - 1))}{k - (\sigma - 1)} \right)^{\frac{1}{\tau}}. \]  

(37)

We see that the cutoff ability under trade is higher than under autarky since the mass of firms declines under trade. In Figure 2 the equilibrium factor allocation in the open economy is compared to the equilibrium factor allocation under autarky.

**Welfare**

Aggregate output in the open economy is given by \( Y = M(1 + \chi)r(\hat{\varphi}). \) Following the same steps as in the derivation of welfare under autarky we get

\[ Y_o^n = M^{\frac{\sigma}{\tau - 1}} \left( \frac{k}{k\sigma - (\sigma - 1)} \right)^{\frac{\sigma}{\tau - 1}} (\sigma - 1)(1 + \chi^n)\frac{\sigma}{\tau - 1} \varphi^*. \]  

(38)

Substituting for the mass of firms \( M \) and the cutoff productivity \( \varphi^* \) using the equilibrium values we find

\[ Y_o^n = (\sigma - 1)\left( \frac{k}{k\sigma - (\sigma - 1)} \right)^{\frac{\sigma}{\tau - 1}} \left( \frac{k\sigma - (\sigma - 1)}{k - (\sigma - 1)} \right)^{\frac{1}{\tau}} N^{\frac{\sigma}{\tau - 1}} (1 + \chi^n)^{\frac{1}{\tau}}. \]  

(39)
Welfare is defined as per-capita income and is therefore given by

\[
\left( \frac{Y}{N} \right)_o^n = (\sigma - 1) \left( \frac{k}{k\sigma - (\sigma - 1)} \right)^{\frac{\sigma - 1}{\sigma}} \left( \frac{k\sigma - (\sigma - 1)}{k - (\sigma - 1)} \right) \frac{1}{k} N^{\frac{1}{k-1}} \left( 1 + \chi^n \right)^{\frac{1}{k}}.
\]

(40)

Also in the open economy equilibrium we see the non-distortionary character of the tax-transfer system. Welfare is not affected by the redistribution scheme. Comparing welfare in the trading equilibrium with welfare under autarky we see that the following relationship holds

\[
\left( \frac{Y}{N} \right)_o^n = (1 + \chi^n)^{\frac{1}{k}} \left( \frac{Y}{N} \right)_a^n.
\]

(41)

It becomes obvious that welfare in the trading equilibrium is higher than welfare in the autarky equilibrium. The reason for this finding is the higher cutoff productivity in the open economy that leads to an efficiency gain for the economy.

Size of the welfare state

We want to see how the size of the welfare state for a given tax rate \( t \) is affected by trade. In the open economy the size of the welfare state is given by \( \text{SizeWelfareState}^n_o = t [M\bar{\pi}_o + Lw + \chi Mf] \). Using all the equilibrium values for the endogenous variable, the size of the welfare state is determined as follows

\[
\text{SizeWelfareState}^n_o = t \left[ (1 + \chi^n)^{\frac{1}{k}} (\sigma - 1) \left( \frac{k}{k\sigma - (\sigma - 1)} N \right)^{\frac{\sigma - 1}{\sigma}} \left( \frac{k\sigma - (\sigma - 1)}{k - (\sigma - 1)} \right)^{\frac{1}{k}} \right].
\]

(42)
Looking at Eq. (20), it becomes obvious that the following relationship holds for a given tax rate $t$

$$\text{SizeWelfareState}_o^n = (1 + \chi^n)^\frac{1}{k}\text{SizeWelfareState}_a^n.$$  \hspace{1cm} (43)

Since $(1 + \chi^n)^\frac{1}{k} > 1$ we can conclude that for a given tax rate $t$ the size of the welfare state in the open economy equilibrium is greater than the size of the welfare state under autarky. Because the population size $N$ is exogenously given, this implies also that the transfer per individual increases when going from the autarky to the open economy equilibrium of the model. This finding should come as no surprise. The non-distortionary tax-transfer system puts a proportional tax on both profit and wage income and we already showed that total income in the economy increases due to trade.

**Income distribution**

We want to see how the income distribution is affected by trade. We start by looking at inter-group inequality. Inter-group inequality is measured by the ratio of the average post tax-transfer profit of domestic firms and post tax-transfer labour income. It can be determined as follows

$$\frac{(1 - t)\bar{\pi}_o + b}{(1 - t)w + b}.$$ \hspace{1cm} (44)

We just argued that the size of the welfare state increases when going from autarky to trade and this implies a higher transfer per capita in the open economy than in the closed economy. It is straightforward to show that ceteris paribus a higher transfer decreases inter-group inequality. In order to establish the overall effect of trade on inequality we first have to solve for inter-group inequality in terms of only exogenous parameters. Noting that average profits of domestic firms in the open economy are given by $\bar{\pi}_o = (1 + \chi)\pi(\tilde{\varphi}) - \chi f$ and using the fact that $\pi(\tilde{\varphi}) = \frac{k}{k-1}\pi(\varphi^*)$ as well as the equilibrium factor allocation and imposing the budget constraint we can calculate inter-group inequality in the open economy as follows

$$\text{InterIneq}_o^n = \frac{(1 - t)\left(1 + \chi^n\frac{k}{k-1} - \chi^n\right) + t\frac{\alpha}{\kappa\sigma - (\sigma - 1)}}{1 - t + \frac{\alpha}{\kappa\sigma - (\sigma - 1)}}.$$ \hspace{1cm} (45)

Comparing inter-group inequality in the open economy with inter-group inequality under autarky it becomes obvious that trade leads to an increase in inter-group inequality. The reason for this finding is as follows. Although trade leads to an increase in both profit income and labour income, profit income increases by more than labour income since there are additional profits generated through exporting. The trade-induced increase in the transfer per capita counteracts this effect, yet cannot outweigh it. It is important to note that trade in fact leads to an increase in average
income for both groups in this economy. However, the gap between average incomes of the two groups in the economy still widens due to trade. It is straightforward to show that \( \frac{\partial \text{InterIneq}}{\partial t} < 0 \). Hence, inter-group inequality decreases in the tax rate.

Within the group of managers inequality is determined by the Gini coefficient. In the open economy we need to distinguish between exporters and non-exporters within the group of managers. We therefore get a Lorenz curve with two segments. The Gini coefficient for managerial income in the open economy with a non-distortionary redistribution scheme in place is given by\textsuperscript{5}

\[
\text{IntraIneq}_n = \frac{(1-t)(1+\chi^n)\frac{k}{k-(\sigma-1)} - (1-t)\chi^n - (1-t)(\chi^n)^2 \frac{\sigma-1}{2k-(\sigma-1)} - (1-t)\frac{2k}{2k-(\sigma-1)}(1-t)\chi^n + t\frac{k\sigma}{k\sigma-(\sigma-1)}}{(1-t)(1+\chi^n)\frac{k}{k-(\sigma-1)} - (1-t)\chi^n + t\frac{k\sigma}{k\sigma-(\sigma-1)}}. \tag{46}
\]

It is straightforward to show that inequality within the group of managers is higher in the trading equilibrium than it is under autarky. As described in Eggert and Kreickemeier (2012) there are two effects that lead to this result. First, due to the exporting fixed cost there is selection of only the most productive firms into exporting. This leads to extra profits of exporting firms relative to non-exporters. Second, all exporters have to pay the same amount of fixed cost. This means that the fixed cost are a higher burden for less productive exporters. The transfer per capita is higher in the open economy than in the closed economy. Ceteris paribus a higher transfer leads to a decrease in intra-group inequality. However, since there is an overall increase in intra-group inequality, we conclude that the effects that increase inequality are stronger. It can also be shown that \( \frac{\partial \text{IntraIneq}_n}{\partial t} < 0 \). This means that intra-group inequality decreases in the tax rate.

For a given tax rate trade leads to more inequality on two fronts: both higher inter-group inequality and higher intra-group inequality. This result is remarkable in the sense that inequality rises although the size of the welfare state and hence the transfer payment increases as well. The tax-transfer system considered is non-distortionary because it does not affect pre tax-transfer income. Therefore it is always possible to decrease trade-induced inequality without reducing the gains from trade. In order to achieve this, trade liberalisation needs to be accompanied with an increase in the tax rate.

\textbf{4 Conclusion}

This paper analyses the effects of international trade in a model with heterogeneous firms where a welfare state redistributes income. Ceteris paribus, international trade increases aggregate income but also leads to higher inequality in two important dimensions. Both inter-group inequality and

\textsuperscript{5}The derivation of both the Lorenz curve and the Gini coefficient is relegated to the Appendix.
intra-group inequality increase due to trade. For a given tax rate the size of the welfare state and therefore the transfer per capita increases with trade. This second-round effect c.p. decreases trade-induced inequality. However it cannot outweigh the primary increase in inequality due to trade. The non-distortionary redistribution scheme does not affect the occupational choice of individuals and therefore leaves pre tax-transfer income unaffected. Hence it is possible to decrease trade-induced inequality without jeopardising the gains from trade by increasing the tax rate. In order for the current level of economic integration to be sustainable “a New Deal for globalisation” is needed as argued by Scheve and Slaughter (2007). This new deal should link trade liberalisation to redistribution policies. Therefore a sound understanding of how globalisation and the welfare state interact is needed to shape globalisation. This paper aims to contribute to this discussion.
References


A Appendix

A.1 Derivation of Eq. (7)

In order to derive average productivity as a function of marginal productivity we note that average revenues are given by

\[ \bar{r} = \frac{1}{1-G(\varphi^*)} \int_{\varphi^*}^{\infty} r(\varphi) dG(\varphi). \]  \hspace{1cm} (A.1)

Using Eq. (6) and \( G(\varphi) = 1 - \varphi^{-k} \) we get

\[ \bar{r} = \frac{k}{k-(\sigma-1)} r(\varphi^*). \]  \hspace{1cm} (A.2)

Again using Eq. (6) together with Eq. (A.2) we get Eq. (7).

A.2 Derivation of Eq. (25)

The cumulative managerial income of all managers with a productivity level lower than or equal to \( \varphi \in [\varphi^*, \infty) \) relative to the aggregate managerial income is given by Eq. (23). In a next step we use the budget constraint of the government and the equilibrium factor allocation. Furthermore noting that \( \Pi = M \pi(\tilde{\varphi}) \), \( \pi(\tilde{\varphi}) = \frac{k}{k-(\sigma-1)} \) and \( M(\varphi) = M \left( 1 - \left( \frac{\varphi^*}{\varphi} \right)^k \right) \) we get

\[ (1-t) \frac{k}{k-(\sigma-1)} \left( \frac{\varphi^*}{\varphi} \right)^{k-(\sigma-1)} - (1-t) \frac{k}{k-(\sigma-1)} + t \frac{k}{k-(\sigma-1)} - t \frac{k}{k-(\sigma-1)} \left( \frac{\varphi^*}{\varphi} \right)^k. \]  \hspace{1cm} (A.3)

Eq. (A.3) together with Eq. (24) deliver the Lorenz curve for post tax-transfer managerial income.

A.3 Deriving Eq. (46)

When deriving intra-group inequality in the open economy we need to distinguish between exporters and non-exporters. The Lorenz curve will have two segments. The ratio of cumulative post tax-transfer managerial income for all non-exporters, \( I_{ne} \), with a productivity level lower than or equal to \( \tilde{\varphi} \in [\varphi^*, \varphi^*] \) and aggregate post tax-transfer managerial income in the open economy, \( I_o \), is given by

\[ \frac{I_{ne}(\tilde{\varphi})}{I_o} = \frac{(1-t) \Pi^{ne}(\tilde{\varphi}) + bM(\tilde{\varphi})}{(1-t) \Pi_o + bM}. \]  \hspace{1cm} (A.4)

Noting that

\[ \Pi^{ne}(\tilde{\varphi}) = \frac{M}{1-G(\varphi^*)} \int_{\varphi^*}^{\varphi^*} \pi(\varphi) dG(\varphi) = \frac{k}{(\sigma-1)-k} M \pi(\varphi^*) \left( \frac{\varphi^*}{\varphi} \right)^{k-(\sigma-1)} - 1, \]  \hspace{1cm} (A.5)

\[ \] \hspace{1cm} \footnote{Note that it is straightforward to show that this ratio increases in the transfer b.}
where we used Eq. (6);

\[
M(\varphi) = \frac{M}{1 - G(\varphi^* )} \int_{\varphi^* }^{\varphi} dG(\varphi) = M \left( 1 - \left( \frac{\varphi^*}{\varphi} \right)^k \right); \quad (A.6)
\]

\[
\Pi_o = M \left[ (1 + \chi) \pi(\varphi) - \chi f \right] \quad (A.7)
\]

and using the budget constraint of the government as well as the equilibrium factor allocation we get

\[
\frac{J_{ne}(\varphi)}{J_o} = \frac{(1 - t)(1 - \gamma)k}{(\sigma - 1)k} - (1 - t)\frac{\gamma}{k} - (1 - t)\frac{k}{(\sigma - 1)k} - t\frac{ka}{k\sigma - (\sigma - 1)} \left( \varphi \right)^k . \quad (A.8)
\]

The ratio of firms with productivity levels lower than or equal to \( \varphi \) is given by

\[
\gamma = 1 - \left( \frac{\varphi}{\varphi^*} \right)^k . \quad (A.9)
\]

Combining Eq. (A.8) with Eq. (A.9) we get the first segment of the Lorenz curve

\[
QI^n_o(\gamma) = \frac{(1 - t)(1 - \gamma)k}{(\sigma - 1)k} - (1 - t)\frac{\gamma}{k} - (1 - t)\frac{k}{(\sigma - 1)k} - t\frac{ka}{k\sigma - (\sigma - 1)} (1 - \gamma) . \quad (A.10)
\]

Evaluating \( QI^n_o \) at \( \gamma = h_M \) with \( h_M \equiv 1 - \left( \frac{\varphi^*}{\varphi} \right)^k = 1 - \chi^a \) yields

\[
QI^n_o(h_M) = \frac{(1 - t)(1 - \gamma)k}{(\sigma - 1)k} (\chi^a)^{1 - \frac{\gamma}{k}} - (1 - t)\frac{\gamma}{k} - (1 - t)\frac{k}{(\sigma - 1)k} - t\frac{ka}{k\sigma - (\sigma - 1)} \chi^a . \quad (A.11)
\]

The ratio of cumulative post tax-transfer managerial income for all firms (exporters and non-exporters) with a productivity level up to \( \varphi \in [\varphi_1^* , \infty) \) and aggregate post tax-transfer managerial income \( I_o \) is given by

\[
\frac{I(\varphi)}{I_o} = QI^n_o(h_M) + \frac{(1 - t)\Pi^e(\varphi) + b \left( M(\varphi) - M(\varphi_1^*) \right)}{(1 - t)\Pi_o + bM} . \quad (A.12)
\]

Note that

\[
M(\varphi^*_1) = \frac{M}{1 - G(\varphi^* )} \int_{\varphi^* }^{\varphi^*_1} dG(\varphi) = M \left( 1 - \left( \frac{\varphi^*_1}{\varphi^*} \right)^k \right) . \quad (A.13)
\]

The exporters profits are given by

\[
\Pi^e(\varphi) = \frac{M}{1 - G(\varphi^* )} \int_{\varphi^* }^{\varphi} \pi^e(\varphi) . \quad (A.14)
\]
With \( \pi^*(\varphi) = \Omega \pi(\varphi) - f \) and by virtue of Eq. (6) we get

\[
\Pi^c(\bar{\varphi}) = \frac{M k \pi(\varphi^*) \Omega}{(\sigma - 1) - k} \left( \frac{\varphi^*}{\bar{\varphi}} \right)^{k-(\sigma-1)} + M \pi(\varphi^*) \left( \frac{\varphi^*}{\bar{\varphi}} \right)^k - M \pi(\varphi^*) \left( \frac{\varphi^*}{\varphi^*} \right)^k. 
\] (A.15)

Using Eqs. (A.6), (A.7), (A.13), (A.15) and taking into account the budget constraint of the government as well as the equilibrium factor allocation Eq. (A.12) becomes

\[
\frac{I(\bar{\varphi})}{I_o} = Q I_o^w(h_M) + \frac{- (1 - t) \frac{k \Omega}{k \sigma - (\sigma - 1)} (\frac{\varphi^*}{\bar{\varphi}})^{\sigma - 1 - k} + (1 - t) (\frac{\varphi^*}{\varphi^*})^{-k} + (1 - t) \frac{k \Omega}{k \sigma - (\sigma - 1)} (\frac{\varphi^*}{\varphi^*})^{\sigma - 1 - k}}{(1 - t)(1 + \chi^n) - (1 - t) \chi^n + t \frac{k \sigma}{k \sigma - (\sigma - 1)}}
+ \frac{-(1 - t) (\frac{\varphi^*}{\bar{\varphi}})^{-k} + t \frac{k \sigma}{k \sigma - (\sigma - 1)} (\frac{\varphi^*}{\varphi^*})^{-k} - t \frac{k \sigma}{k \sigma - (\sigma - 1)} (\frac{\varphi^*}{\varphi^*})^{-k}}{(1 - t)(1 + \chi^n) - (1 - t) \chi^n + t \frac{k \sigma}{k \sigma - (\sigma - 1)}}. 
\] (A.16)

Substituting \( \gamma = 1 - (\frac{\varphi^*}{\bar{\varphi}})^{-k} \) and \( h_M \equiv 1 - (\frac{\varphi^*}{\varphi^*})^{-k} = 1 - \gamma^n \) we get the second segment of the Lorenz curve

\[
Q I_o^w(\gamma) = Q I_o^w(h_M) + \frac{- (1 - t) \frac{k \Omega}{k \sigma - (\sigma - 1)} (1 - \gamma) (\frac{\varphi^*}{\bar{\varphi}})^{k - 1} + (1 - t)(1 - \gamma) + (1 - t) \frac{k \Omega}{k \sigma - (\sigma - 1)} (\chi^n)^{k - 1}}{(1 - t)(1 + \chi^n) - (1 - t) \chi^n + t \frac{k \sigma}{k \sigma - (\sigma - 1)}}
+ \frac{-(1 - t) \chi^n + t \frac{k \sigma}{k \sigma - (\sigma - 1)} \chi^n - t \frac{k \sigma}{k \sigma - (\sigma - 1)} (1 - \gamma)}{(1 - t)(1 + \chi^n) - (1 - t) \chi^n + t \frac{k \sigma}{k \sigma - (\sigma - 1)}}. 
\] (A.17)

with \( Q I_o^w(h_M) = Q I_o^w(h_M) \).

Putting the two segment together, the Lorenz curve in the open economy with the non-distortionary redistribution scheme in place can be written as

\[
Q I_o^w(\gamma) = \begin{cases} 
Q I_o^w(\gamma) & \text{if } \gamma \in [0, h_M) \\
Q I_o^w(\gamma) & \text{if } \gamma \in [h_M, 1].
\end{cases}
\] (A.18)

The Gini coefficient for the post-tax-transfer managerial income distribution can then be calculated as

\[
\text{IntraIneq}^w_{o} = 1 - 2 \int_0^1 Q I_o^w(\gamma) d\gamma = 1 - 2 \left[ \int_0^{h_M} Q I_o^w(\gamma) d\gamma + \int_{h_M}^1 Q I_o^w(\gamma) d\gamma \right]. 
\] (A.19)

Using Eqs. (A.10) and (A.17) and following tedious but straightforward calculation we get Eq. (46).