and B. Kämpfer

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aResearch Center Rossendorf, Institute for Nuclear and Hadron Physics, PF 510119 01314, Dresden, FRG
bLab.Theor. Phys., Joint Institute for Nuclear Research, Head Post Office, P.O. Box 79, Moscow, Russia
cTheoretical Physics Institute, University of Alberta, Edmonton, Alberta T6G 2J1, and
TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C. V6T 2A3, Canada
dInst. Theor. Phys. (KAI e. V.), Technical University, Mommsenstraße 13, Dresden, FRG

Abstract

A consistent theoretical approach is suggested for the description of deep inelastic scattering of polarized leptons off polarized deuterons within the operator product expansion method and an effective meson-nucleon theory. Our approach describes fairly well recent deuteron experimental data on the spin structure function $g_1(x)$.

1 Introduction. One of the topical problems of the present experimental and theoretical investigation of deep inelastic scattering (DIS) of leptons from hadronic targets is the determination of the electromagnetic and weak interaction characteristics of the neutron. The proton and neutron properties together will allow one to verify sum rules of Quantum Chromodynamics and predictions of QCD-motivated models. In previous experiments some indications of a breakdown of the fundamental symmetries are found (e.g., the famous spin crisis [1], the isospin symmetry violation [2, 3]), and the validity of both the Bjorken and Gottfried sum rules are called into question. The new experiments, starting now at DESY [4], SLAC [5] and by SMC [6] are aimed to clarify this subject.

The SM [6] and E142 [5] Collaborations have recently obtained first results on the polarisation asymmetry of the cross sections and the spin dependent structure functions (SSF). These data together with earlier result of the EM Collaboration [1] can now be used for consistency checks. The SLAC data on polarized $^3$He [5], after the subtraction of nuclear structure effects [7, 8], one can extract the neutron SSF and compute the Bjorken sum rule integral. It is found that Bjorken sum rule is slightly violated (by about three standard deviations), whereas the first moment of the neutron SSF is found to be in a good agreement with the Ellis-Jaffe sum rule. This result seems to be in conflict with the conclusions drawn from the EMC experiments [1], that is, there is no room for the spin crisis in the SLAC results [5]. At the same time, the SM Collaboration performs the first measurements on polarized deuterons [6]. From these data one can estimate the SSF of the "isoscalar" nucleon.

1On leave from Far Eastern State University, Vladivostok, Russia
Combining these data with EMC proton data one finds the validity of the Bjorken sum rule. Note the rather large errors (\( \sim 100\% \)) in the SMC data. It is worth stressing here that the SLAC and SMC measurements are performed with nuclear targets, and the interpretation of the data requires an accurate theoretical model describing the DIS off polarized nuclei. Only after a correct subtraction of the nuclear structure effects, a discussion about self consistency of data interpretations can be meaningful.

In the previous papers [9, 10] we suggest a theoretical model to analyze the DIS off unpolarized targets by using the operator product expansion method within an effective meson nucleon theory with one-boson-exchange interaction. Within this approach a good description of DIS off unpolarized nuclei is achieved in a large interval of nuclear masses and the Bjorken scale variable \( x \). The purpose of the present paper is to extend the proposed model to the polarized processes of DIS off a deuteron target. We apply our new approach to the SSF of the deuteron and find good agreement with recent data.

2 Basic Formalism. The SSF’s are determined experimentally by measuring the asymmetry in the reaction with polarized particles \( \vec{l} + \vec{A} \rightarrow l' + \text{anything} \). The antisymmetrical part of the cross section for inelastic lepton scattering, \( W_{\mu \nu} \), is proportional to the imaginary part of the corresponding amplitude \( T_{\mu \nu} \) for forward, virtual Compton scattering from the polarized target,

\[
W_{\mu \nu}(Q^2, S) = \epsilon_{\mu \nu \lambda \sigma} q^\lambda \left[ S^\sigma G_1(q_0, Q^2) + ((p \cdot q) S^\sigma - (S \cdot q)p^\sigma) \frac{G_2(q_0, Q^2)}{M_A^2} \right] = \frac{1}{2\pi} \text{Im} T_{\mu \nu}(Q^2, S). \tag{1}
\]

Here \( G_{1,2} \) are the spin-dependent structure functions, \( p^\mu = (M_A, 0) \) denotes the four momentum of the target, \( M_A \) stands for the mass of the target. \( S \) is the normalized spin four vector \(( S \cdot S = -1, S \cdot p = 0) \). The transferred momentum is \( Q^2 \equiv -q^2 \), where in the laboratory frame \( q^\mu = (\nu, 0, 0, -|q|) \).

In the Bjorken limit \((-Q^2 \rightarrow \infty, \nu \rightarrow \infty, x_A \equiv q^2/2M_A \nu - \text{fixed})\) the functions \( G_{1,2} \) are predicted to depend only upon \( x_A \), yielding the scaling spin dependent structure functions, \( g_1(x_A) = \nu G_1(\nu, Q^2) \) and \( g_2(x_A) = \nu^2 G_2(\nu, Q^2)/M_A \). Our further consideration will be performed in the Bjorken limit.

The Compton amplitude \( T_{\mu \nu} \) is given by a matrix element of the time-ordered product of two electromagnetic current operators with the polarized target ground state \( |p, H\rangle \), where \( H \) denotes the projection of target spin on the \( z \)-axis. Since in DIS kinematics the transferred momentum \( q^2 \) is large, the main contribution to the amplitude \( T_{\mu \nu} \) comes from small space-time intervals or, consequently, from the singularities of the product of operators with the coinciding arguments. The most reliable method of analyzing these singularities is the Wilson’s operator product expansion (OPE) [11, 12, 13]. Within it the product of two operators is expanded in sets of local operators with increasing order of their twist. The lowest
values of the twist give the leading contribution in the Wilson's series.

In our case the OPE of the antisymmetric part of Compton amplitude in the leading twist order reads

\[ T_{\mu\nu} = i\epsilon_{\mu\nu\lambda\sigma}q^\lambda \sum_{t;n=m_0,2,\ldots}^{\infty} C_{n,t}(Q^2) \left( \frac{2}{Q^2} \right)^{n+1} q_{\mu_1} \ldots q_{\mu_n} \langle p, H | \hat{O}_t^{\sigma(\mu_1 \ldots \mu_n)}(0) | p, H \rangle, \]  

(2)

where \( t \) stands for the fundamental fields of the theory under consideration, \( \{ \mu\nu \ldots \} \) denotes the symmetrization in the Lorentz indices, \( C_{n,t} \) are the Wilson's coefficients (being \( c \)-numbers), and \( \hat{O}_t^{\sigma(\mu_1 \ldots \mu_n)}(0) \) denotes the set of twist-two operators\(^2\). The explicit form of these operators is to be constructed from the field operators involved into the consideration. For instance, for the nucleon spinor fields \( N_i(0) \) these operators are

\[ \hat{O}_t^{\sigma(\mu_1 \ldots \mu_n)}(0) = \left( \frac{i}{2} \right)^n : \bar{N}_t(0) \gamma^\nu \gamma_5 \partial^{\mu_1} \ldots \partial^{\mu_n} N_i(0) :. \]  

(3)

Note that in eq. (2), due to the factorization property of the OPE, the Wilson coefficient functions \( C_n \) are related to short-distance physics ("subnucleonic" physics), whereas the matrix elements of operators \( \hat{O}_t^{\sigma(\mu_1 \ldots \mu_n)} \) characterize the large-distance physics (or effects of nuclear structure). We are interested in the investigation of the role of nuclear structure in DIS, hence further the main attention will be paid to these operators \( \hat{O}_t^{\sigma(\mu_1 \ldots \mu_n)} \).

For our analysis the use of helicity amplitudes is convenient. The helicity amplitudes \( A_{\lambda H, \lambda' H'} \), \( h_{\lambda H, \lambda' H'} \) for a process \( \gamma_\lambda + \text{target}_H = \gamma_{\lambda'} + \text{target}_{H'} \), where \( \lambda, \lambda' \) and \( H, H' \) are the spin components of the photon and target, respectively, along the \( z \)-axis are defined as follows

\[ A_{\lambda H, \lambda' H'} = \epsilon_\lambda^{\mu \nu} W_{\mu \nu}(S_{HH'}) \epsilon_{\lambda'}^\nu, \quad h_{\lambda H, \lambda' H'} = \epsilon_\lambda^{\mu \nu} T_{\mu \nu}(S_{HH'}) \epsilon_{\lambda'}^\nu. \]

Here \( \epsilon_\lambda^\mu \) is the polarization vector of an helicity \( \lambda \) photon, i.e., \( \sqrt{2} \epsilon_\lambda^\mu = \mp (0,1,\pm i,0) \), \( \epsilon_\lambda^0 = (-q_z,0,0,q_0)/\sqrt{-q^2} \), the other notation is obvious. The scaling structure function \( g_1 \) is completely determined by two helicity amplitudes

\[ g_1 = -\frac{1}{2} (A_{+++} - A_{++-}) = -\frac{1}{4\pi} \text{Im} [h_{+++} - h_{+-+}]. \]  

(4)

The contribution of the operators \( \hat{O}_t^{\sigma(\mu_1 \ldots \mu_n)} \) (3) to the Compton helicity amplitude is of the form

\[ h_{+H, +H} = - \sum_{t;n=0,2,\ldots}^{\infty} C_{n,t}(-q^2) \left( \frac{2q_0}{-q^2} \right)^{n+1} \langle \hat{O}_t^{+}(n) \rangle_A, \]

\[ \langle \hat{O}_t^{+}(n) \rangle_A = \left( \frac{i}{2} \right)^n \langle p_A H | : \bar{N}_t(0) \gamma^+ \gamma_5 \partial_+^n N_i(0) : | p_A H \rangle, \]  

(5)

\(^2\)Note that in eq. (2) there is no definite symmetry of the operator \( \hat{O} \) relative to the index \( \sigma \). An originally symmetric operator \( \hat{O} \) with indices \( \{ \sigma \mu_1 \ldots \mu_n \} \) has been rearranged to exhaust the so-called Wandzura-Wilczek term [14]. This term is relatively small and contributes to the SSF \( g_2(x) \) only.
where $\gamma_+ = \gamma_0 + \gamma_z$ and $\partial_- = \partial_0 - \partial_z$. In deriving eq. (5) we use that fact that in DIS kinematics $|\vec{q}| \approx q_0$.

Eqs. (3)-(5) show the necessity in the analysis of the SSF within OPE method to rely on a consistent determination of both the Wilson coefficient functions $C_{n,t}$ and the matrix elements of the twist-two operators $\hat{O}^{\sigma(m_1 \ldots m_n)}$ sandwiched between nuclear ground state vectors $|n A H\rangle$. So far there does not exist a rigorous theoretical method for computing simultaneously both these parts. Actually only one of them can be calculated in a self-consistent way, the other remains to be fixed from experiment. We are interested in studying the nuclear effects in DIS, i.e., in the computation of the nuclear matrix elements of the operator $O^{\sigma(m_1 \ldots m_n)}$.

For this one needs a field theory within that the nuclear ground state vectors and other characteristic of a nucleus, such as binding energy, interacting potential, wave function etc., are well described. The explicit form of the operators $\hat{O}^{\sigma(m_1 \ldots m_n)}$ are then determined. Here we apply an effective meson-nucleon theory of $NN$ forces to the OPE method. First we choose the interaction Lagrangian with pseudoscalar isovector coupling, that is the $\pi NN$ vertex with coupling strength $g_{\pi NN}$, $L_{\text{int.}} = -i g_{\pi NN} \sum_{t,t',\alpha} \bar{N}_t(x) \gamma_5 \gamma_{t'} \phi_{\alpha}(x) N_{t'}(x)$, where $\alpha = x, y, z$ and $t, t' = 1, 2$ stand for isospin indices, $\phi$ is the isovector pion field. This Lagrangian belongs to the renormalized theories, so that the application of OPE here is justified [11, 12].

The spinor fields are the fundamental ones of the theory, i.e., the protons and neutrons here are considered as "bare" nucleons. They appear to be point-like particles, and the scattering off them has to be connected in OPE with the Wilson's coefficients. The physical nucleons ought to be "dressed" by the interacting mesonic clouds and, consequently, the nuclear target has to be presented as a superposition of bare nucleons with mesonic cloud and mesonic exchanges. The most natural way of dressing the hadron target is provided by the Tamm-Dancoff method, according to that the physical ground state of a nucleus may be represented as

$$|A\rangle_{PA,ts} = \sqrt{1 - Z_A \varphi_0 |\vec{N}_1, \vec{N}_2, \ldots, \vec{N}_A\rangle_{PA,ts} + \varphi_1 |\vec{N}_1, \vec{N}_2, \ldots, \vec{N}_A, \pi\rangle_{PA,ts} + \ldots,}$$  \hspace{1cm} (6)$$

where $Z_A$ stands for the renormalization constant, $s$ is the spin index. The functions $\varphi_i$ are determined by the Hamiltonian of the system and the condition that the wave function (6) describes the nuclear ground state [15]. The further procedure of computing the matrix elements of the Compton amplitude is the following one: using the Hamiltonian of the system, that is the equation of motion for the fields in the Heisenberg representation, one computes the explicit form of the operators (5); then by making use of the expansion (6) the matrix elements are calculated. Obviously, the resulting matrix elements contain some divergent integrals, related to the self-energy like corrections to the bare nucleons, i.e. the "dressing" diagrams. The same integrals appear when computing matrix elements for the free physical nucleons. Since we investigate the effects of nuclear structure, these divergent parts must be subtracted from the nuclear matrix elements, so that the finite results depend
only on characteristics of physical nucleons and nuclear structure. The Wilson’s coefficients \( C_n \), which are target independent, are the same in both cases and determine the DIS off the bare constituents and ought to be included into the matrix elements of the physical nucleons.

3 Moments of the SSF of the Deuteron. As it has been mentioned above one of the conditions of self-consistency of the present approach is that the Hamiltonian of the system and the Tamm-Dancoff expansion (6) describe the main features of the target. It is known [16] that in nonrelativistic limit the effective meson-nucleon theory and Tamm-Dancoff method result in one-boson-exchange potential that gives a good description of the deuteron. The procedure of nonrelativistic reduction of classical equation of motion for the interacting mesonic and nucleonic fields is well established, and details (e.g., \( Z_D, \varphi_i \)) can be found in refs. [15]. Making use of the nonrelativistic nucleon fields and nonrelativistic Hamiltonian of the system we compute the explicit form of the operators \( \hat{O}_{\pi}^{(\mu_1 \cdots \mu_m)} \) and the corresponding matrix elements for the physical nucleon and deuteron. After some cumbersome algebra and the subtraction of the divergent parts, coming from the nucleon matrix elements, the moments of the deuteron SSF may be written as

\[
\frac{1}{2} \left( \frac{M_D}{m} \right)^n M_{n+1}^D(g_1) = M_{n+1}^N(g_1) \int \frac{dp}{(2\pi)^3} \Delta f^{I,A}(p) \left( 1 + \frac{p_z}{m} + \frac{p^2}{2m^2} \right)^n + \\
M_{n+1}^N(g_1) \int \frac{dpdk}{(2\pi)^6} \Delta f^{int}(p,k) \frac{1}{k_z} \left[ \left( 1 + \frac{k_z}{2m} \right)^n - \left( 1 - \frac{k_z}{2m} \right)^n \right], \tag{7}
\]

where \( n = 0, 2, \ldots \), \( M_{n+1}^N \) stands for the moments of the "isoscalar" physical nucleon, \( M_{n+1}(g_1) = \int dx x^ng_1(x) \). The definition of \( \Delta f \) is \( \Delta f = \frac{1}{2}(f_{+1+1} - f_{-1-1}) \), and

\[
f_{I,H}^{I,A}(p) = \Psi_H^D(p) + \left( \frac{S \cdot p}{m} + S_z + \frac{S \cdot p}{2m^2} p_z \right) \Psi_H^N(p), \tag{8}
\]

\[
f_{I,H}^{int}(p,k) = \Psi_H^D(p) + \frac{1}{2} (S_z V_\pi(k) + V_\pi(k) S_z) \Psi_H^N(p + k) \tag{9}
\]

where \( S \) is the operator of the total spin of the nucleons and \( V_\pi(k) \) denotes the one-boson-exchange potential. The first term in eq. (7) is usually referred to as the impulse approximation contribution, while the second one is of the pure interaction origin and reflects the influence of boundness of the nucleons inside the deuteron.

Applying the inverse Mellin transform to eq. (7) we reconstruct the deuteron structure function \( g_1^D \) in the convolution form

\[
\frac{1}{2} g_1^D(x) = \int \frac{dy}{y} \frac{dy}{y} g_1^N \left( \frac{x}{y} \right) \left( \Delta f_{I,A}(y) + \Delta f_{int}(y) \right), \tag{10}
\]

where the distribution functions \( \Delta f_{I,A}(y) \) and \( \Delta f_{int}(y) \) are given by

\[
\Delta f_{I,A}(y) = \int \frac{dp}{(2\pi)^3} \Delta f^{I,A}(p) \delta \left( y - 1 - \frac{p_z}{m} - \frac{p^2}{2m^2} \right), \tag{11}
\]

5
\[ \Delta f_{\text{int.}}(y) = \int \frac{dp\,dk}{(2\pi)^6} \Delta f_{\text{int}}(p, k) \frac{1}{k_z} \left[ \delta \left( 1 - y + \frac{k_z}{2m} \right) - \delta \left( 1 - y - \frac{k_z}{2m} \right) \right] \theta(y). \]

Equations (7), (10) and (11) are the basic results in the determination of the deuteron moments and the deuteron SSF within the operator product expansion approach with one-boson-exchange. The expression for the impulse approximation contribution practically coincides with one used earlier [7] for estimates of the first moments of the deuteron SSF. For the first moment it gives almost the same result as obtained in ref. [7], i.e., \( M_1^D \approx (1 - 3/2P_D)M_1^N \), where \( P_D \) is the probability of the \( D \)-wave admixture in the deuteron. The distribution function \( \Delta f_{\text{IA.}}(y) \) is interpreted as the distribution of the constituents with spins up and down having a kinetic energy \( p^2/2m^2 \) inside the up-polarized deuteron. Formulae (10) and (11) become more compact if one substitutes in eq. (11) the difference of the two \( \delta \)-functions by its first derivative

\[ \frac{1}{2} g_1^D(x) \approx g_1^{A^*}(x) + \left[ g_1^N(x) + x \frac{d}{dx} g_1^N(x) \right] \langle \{S_z, V_{\pi}\} \rangle_D. \] (12)

The second term in eq. (12) is the correction to the impulse approximation due to the boundness of the nucleons. This contribution is rather small (\( \sim (1 - 3/2P_D)(V_{\pi})/m \)) and essentially depends on the behavior of the nucleonic structure function \( g_1^N(x) \) and its first derivative.

Formulae (7) - (12) have been obtained for the pseudoscalar isovector coupling. The deuteron wave function \( \Psi^D(p) \) in this case is the solution of the Schrödinger equation with the pion-nucleon interaction potential \( V_{\pi}(k) \). Obviously, this wave function and potential are not yet sufficient to describe the properties of the physical deuteron. For a thorough analysis it is necessary to take into account other mesons contributing to the one-boson-exchange potential, i.e., the \( \sigma, \omega, \rho, \eta, \) and \( \delta \) mesons that describe well the deuteron [16]. Our calculations with these mesons show that the form of eqs. (7) - (12) is unchanged, except that now the wave function \( \Psi^D(p) \) is the solution of the Schrödinger equation with the full one-boson-exchange potential \( V_{\pi\omega\rho\eta\delta} \).

**4 Results.** For explicit numerical calculation of the deuteron SSF \( g_1^P(x) \) one needs a suitable parametrization of the isoscalar nucleon SSF \( g_1^N(x) \), cf. eqs. (10) and (12). In principle, now there exist experimental data on the proton [1] and the neutron [5] structure function \( g_1(x) \). But they are not yet fully complete, especially at very small \( x \). In this interval some assumptions about the behavior of the nucleon structure function are inevitable. Moreover, the choice of the isoscalar structure function \( g_1^N(x) \) determines whether the Bjorken and Ellis-Jaffe sum rules will be fulfilled or not. We chose here the parametrization of ref. [17] which describes quite well the EMC data on proton.

Figure 1 displays the ratio \( g_1^P/g_1^N \) which illustrates the role of deuteron structure in polarized DIS. The dashed curve is the result of a calculation with only the impulse approximation contribution, while the full curve demonstrates the boundness effects in the deuteron.
Fig. 1: The ratio of the deuteron and isoscalar nucleon spin structure functions. Full line - the contribution of impulse approximation + contribution of bound nucleons to the deuteron SSF; dashed line - the contribution of impulse approximation.

In the interval $0.05 < x < 0.75$ the ratio is almost constant and about 7% less than unity. This is just the effect of destructive contributions of the $D$-wave admixture in the polarized deuteron, i.e., while the deuteron is polarized along the $z$-direction the nucleons may have a spin in the opposite direction due to their orbital ($L = 2$) motion. In contrast to the EMC-effect on unpolarized DIS, the effect of boundness in the polarized case is rather small. This is an understandable effect since in unpolarized case the structure function $F_2(x)$ determines the energy-momentum conservation law, whereas in the polarized case the SSF $g_1^D$ reflects the distribution of polarization of the constituents inside a polarized nucleus. Figure 2 displays our absolute values of the deuteron structure function and the comparison with the SMC experimental data [6]. In spite of rather large errors of these data the comparison with our calculation shows a good agreement. The numerical estimate of the first moment of $g_1^D(x)$ within our approach, $\int dx g_1^D(x) \approx 0.03$, is also in a good agreement with the experimental result $\int dx g_1^{D(SMC)}(x) = 0.023 \pm 0.02 \pm 0.015$ [6].

From these results we conclude that the proposed approach describes the peculiarities of the deuteron spin dependent structure function. Being obtained in a rather consistent way, the model may to be used as a nuclear model for subtractions the nuclear structure effects in extracting the neutron (proton) SSF from combined data of SMC, EMC, SLAC groups (cf. ref. [18] for the method).
5 Summary. In summary, we propose an extension of our theoretical method for investigating the role of the deuteron structure in DIS with polarized particles. We find good agreement with the recent SMC data. Encouraged by this, our approach will be applied to the extraction of the neutron SSF from the combined SMC and EMC experimental data in subsequent work. This would allow for a consistent check of the Bjorken and Ellis-Jaffe sum rule and spin crisis.

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