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Selforganization in the Nuclear System

II: Formation of a New Order
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Abstract

The information entropy of the atomic nucleus is calculated from the wavefunctions of the resonance states. The transition from low to high level density is traced as a function of the coupling strength between the discrete nuclear states and the environment of decay channels. In the critical region of the coupling strength where a redistribution inside the nucleus takes place, information entropy in relation to the discrete states of the closed system is created. Beyond the critical value, a few relevant short-lived modes exist together with long-lived noise. This result is in full correspondence to the maximum information entropy principle of synergetics formulated by Haken. The noise is characterized by disorder expressed by a large information entropy while the relevant modes have a high order and take, correspondingly, a small part of the information entropy of the whole system. The entropy excess accompanying the formation of the new order is used inside the system for creation of noise. Further, the noise is not structureless and the corresponding information entropy is smaller than its maximal value.

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1 Introduction

In our first paper [1] on selforganization in the nuclear system, we showed that the transition from the regime of low to high level density in the open quantum mechanical nuclear system occurs in accordance with the slaving principle of synergetics [2]. Beyond some critical value of the coupling strength between bound and unbound states, stable and unstable modes exist in the system simultaneously. The unstable modes are relevant at time and energy scales characteristic of the system. They correspond to the unstable modes which are shown to determine the behaviour of selforganizing systems [2]. The stable "trapped" modes, on the other hand, are suggested to correspond to the stable "slaved" modes. They are not characteristic of the system and are relevant only at the long-time scale.

The behaviour of an open many-particle system is governed not only by the slaving principle, but also by the maximum information entropy principle as has been shown by Haken [2]. This principle is shown to pass over into the second law of thermodynamics for vanishing coupling to the environment [2]. The validity of the principle of maximal information entropy for an open quantum system is not investigated up to now in a microscopical quantum mechanical approach.

The information entropy of a closed many-particle quantum system is investigated in a few papers [3 - 9] since it can easily be calculated from the spectroscopic values. Although the wavefunctions of the states and hence the information entropy depend on the basic set of wavefunctions and on the size of the configurational space in, e.g., the shell model approach, it is possible to draw some general conclusions from these values in the same manner as from the spectroscopic values. Most interesting is the question whether the information entropy increases as a function of a certain increasing parameter. Such a parameter is, e.g., the strength of the coupling between the basic states which are supposed to have good single-particle quantum numbers. The physical eigenstates are mixed, usually, due to internal as well as external coupling. In this sense, the information entropy represents a measure for the complexity of the eigenstates of the many-body system which have definite total quantum numbers but mostly cannot be characterized by single-particle quantum numbers. In other words, the information en-
entropy characterizes the degree by which the original single-particle quantum numbers of a state of the many-particle system cease to be good quantum numbers. The connection between the complexity of states and quantum chaos is discussed by Haake [8].

In an open quantum mechanical system, the spectroscopic values are complex [10]. It is necessary, therefore, to find an adequate definition for the information entropy which resembles a measure for the complexity of the states of the system also in this case.

In the present paper, we calculate the information entropy for the open nuclear system and check the validity of the maximum information entropy principle. The model used is the continuum shell model described in [1, 10]. The information entropy is defined in Sect. 2. In Sect. 3, the results of numerical calculations are given and discussed in Sect. 4. Conclusions are drawn in the last Section.

2 Model

The model used in the present calculations is the continuum shell model described in [1, 10].

The spectroscopic values follow from

\[(H_{QQ}^{\text{eff}} - \tilde{\varepsilon}_R) \tilde{\Phi}_R = 0\]  \hspace{1cm} (1)

where the \( \tilde{\Phi}_R \) are the complex energy dependent eigenfunctions and

\[\tilde{\varepsilon}_R(E) = \tilde{E}_R(E) - \frac{i}{2} \tilde{\Gamma}_R(E)\]  \hspace{1cm} (2)

are the complex energy dependent eigenvalues of the effective Hamiltonian

\[H_{QQ}^{\text{eff}} = QH (Q + G_P^{(+)} PHQ)\]  \hspace{1cm} (3)

in the subspace of bound states. In (3), \( G_P^{(+)} \) is the Green function for the motion of the particle in the continuum. Further, \( Q \) projects onto the
subspace of bound state wavefunctions,

\[ Q = \sum_{R} |\Phi_{R}^{SM}><\Phi_{R}^{SM}|, \quad (4) \]

and \( P \) onto the environment (subspace of channel wavefunctions) with the completeness relation

\[ P + Q = 1. \quad (5) \]

The bound states \( \Phi_{R}^{SM} \) are identified with the solutions of the standard shell model problem, which reads

\[ (QHQ - E_{R}^{SM})\Phi_{R}^{SM} = 0. \quad (6) \]

The Hamiltonian of the system is \( H = H_0 + V \) with the residual interaction \( V^{in} = \alpha^{in} \cdot V \) in eq. \( (6) \), and \( V^{ex} = \alpha^{ex} \cdot V \) in \( PHQ, \ QHP \) and in the Green function \( G_{P}^{(+)} \), eq. \( (3) \). The parameter \( \alpha \) is used in the calculations as a control parameter by means of which the behaviour of the system is investigated. \( \alpha = 1 \) to 2 corresponds to a realistic value of the residual interaction \( V \).

The basic wavefunctions are the wavefunctions \( \Phi_{i}^{(0)} \) (Slater determinants) of the unperturbed discrete states with all \( A \) particles in bound states, as well as the wavefunctions \( \chi_{c}^{E} \) of the unperturbed channels with \( A - 1 \) particles in bound states and 1 particle in a scattering state. In the \( \Phi_{i}^{(0)} \), the single-particle quantum numbers are good quantum numbers while in the \( \chi_{c}^{E} \), the channel quantum numbers are well defined, i.e. the single-particle quantum numbers of the unbound particle and the total quantum numbers of the corresponding discrete state of the residual nucleus.

The wavefunctions \( \Phi_{R}, \Phi_{R}^{SM} \) and \( \Phi_{R}^{(0)} \) are related by

\[ \tilde{\Phi}_{R} = \sum_{R'} \beta_{RR'}\Phi_{R'}^{SM} \quad (7) \]
with complex coefficients $\beta_{RR'}$ and
\[
\Phi^{SM}_R = \sum_{R'} b_{RR'} \Phi^{(0)}_{R'}
\] (8)

with real coefficients $b_{RR'}$. Further, it is
\[
\Phi_R = \sum_{R'} \eta_{RR'} \Phi^{(0)}_{R'}
\] (9)

with complex coefficients
\[
\eta_{RR'} = \sum_{R''} \beta_{RR''} b_{R''R'}
\] (10)

In order to characterize the mixing of the wavefunctions $\Phi_R$ and $\Phi^{SM}_R$ in relation to the set of basic wavefunctions $\Phi^{(0)}_R$ or $\Phi^{SM}_R$, the values
\[
I^R_\beta = -\sum_{i=1}^N |\beta_{Ri}|^2 \ln |\beta_{Ri}|^2
\] (11)

and
\[
I^R_\eta = -\sum_{i=1}^N |\eta_{Ri}|^2 \ln |\eta_{Ri}|^2
\] (12)

are calculated for every state $R$ in analogy to the value
\[
I^R_\delta = -\sum_{i=1}^N |b_{Ri}|^2 \ln |b_{Ri}|^2
\] (13)

of a closed system. Every $I^R$ characterizes the mixing of the wavefunction of the corresponding state $R$ in relation to the chosen basic set of $N$ wavefunctions. The coefficients $\beta_{Ri}$ and $\eta_{Ri}$ are normalized to 1 by using the definitions
\[
|\beta_{Ri}|^2 = \frac{|\beta_{Ri}|^2}{\sum_i |\beta_{Ri}|^2}
\] (14)
\[ |\eta_{Ri}|^2 = \frac{|\eta_{Ri}|^2}{\sum_i |\eta_{Ri}|^2}. \] (15)

Here, the different \( I^R \) have the following meaning:

\( I_b^R \) characterizes the mixing of the discrete shell model wavefunctions \( \Phi^{SM}_R \) in relation to the basic set of wavefunctions given by the Slater determinants \( \Phi^{(0)}_R \). It is caused by the so-called internal mixing of the shell-model states which is calculated from the residual interaction \( V^{in} = \alpha^{in} \cdot V \) [1].

\( I_\beta^R \) characterizes the mixing of the resonance wavefunctions \( \Phi_R \) in relation to the basic set of shell model wavefunctions \( \Phi^{SM}_R \). It is caused by the so-called external mixing of the shell model states via the continuum which is calculated from the residual interaction \( V^{ex} = \alpha^{ex} \cdot V \) [1].

\( I_\eta^R \) characterizes the mixing of the resonance wavefunctions \( \Phi_R \) in relation to the basic set of wavefunctions given by the Slater determinants \( \Phi^{(0)}_R \). It is caused by both the internal and external mixing.

The sums

\[ I = \sum_R I^R \] (16)

will be considered, in the following, as the information entropies of the system. Here, the sum runs over all \( N \) states

\[ I_\beta = \sum_{R=1}^{N} I^R_\beta \] (17)

or is restricted to the \( K \) relevant states (if such special states exist)

\[ I_\beta = \sum_{R_f=1}^{K} I^{R_f}_\beta. \] (18)

The maximal value of \( I \) is \( I^{max} = N \cdot \ln N \), and \( K \cdot \ln K \), respectively.
3 Results

The calculations are performed for 70 and 190 states, respectively, $J^\pi = 1^-$ of $^{16}O$ and the two open decay channels $^{15}N_{gs} + p$ and $^{15}N^* + p$ in the same manner as described in [1]. Here, $^{15}N_{gs}$ and $^{15}N^*$ are the ground state $J^\pi = \frac{3}{2}^-$ and the first excited state $J^\pi = \frac{1}{2}^-$, respectively, of $^{15}N$. The results are considered as a function of the parameter $\alpha^{ex}$ of the coupling strength between bound and scattering states. We are mostly interested in the degree of mixing $I_{R}^R$ of the wavefunctions $\Phi_R$ which is caused by the external mixing of the shell model states $\Phi_{SM}^R$ via the continuum of decay channels. The $\Phi_{SM}^R$ are the wavefunctions of the states of a closed quantum system and are assumed usually to contain, together with the corresponding eigenvalues $E_{SM}^R$, the spectroscopic information of the states $R$.

The dependence of widths $\bar{I}_R$ of all 70 states on the control parameter $\alpha^{ex}$ is shown in Fig. 1. The widths are measurable values in contrast to the $I_{R}^R$. At a certain value $\alpha_{cr}^{ex}$, a redistribution in the nucleus takes place as a consequence of which the widths of two states (fast relevant modes) become much larger as the widths of all the other ones (trapped modes). The picture shows, further, that the redistribution at $\alpha_{cr}^{ex} \approx 2.6$ is not the only one. At higher values of $\alpha^{ex}$ further redistributions take place by which broad modes of the second and third generation are created. Thus, the trapped modes are not independent from each other, but are correlated. These results illustrate the slaving principle holding in selforganizing systems. They are discussed in [1].

In Fig. 2, the mixing $I_{R}^R$ of the wavefunctions $\Phi_R$ of all 70 states in relation to the basic set $\{\Phi_{SM}^R\}$ is shown as a function of the coupling strength $\alpha^{ex}$. The $I_{R}^R$ of the two fast modes increase at $\alpha_{cr}^{ex}$ strongly. It is, however, impossible to distinguish them from the other modes only on the basis of the behaviour of $I_{R}^R$ although their mixing coefficients increase stronger than those of most of the trapped modes.

It is necessary, therefore, to take into account additionally the information on the lifetime of the states (Fig. 1). In Fig. 3, the $<I_{R}^\sigma>$ averaged over the two fast modes and $<I_{R}^\delta>$ averaged over the 68 trapped modes are
shown. As a result, the $< I^f_\beta >$ increase, in the region $\alpha_{cr}^{ex}$, much stronger than the $< I^b_\beta >$.

In Fig. 4, the information entropy $I_\beta$ is shown as a function of the coupling strength $\alpha^{ex}$. It can be seen easily that $I_\beta$ increases with increasing $\alpha^{ex}$ for all $\alpha^{ex}$ if the sum runs over all 70 states or over the 68 trapped modes. This result is in full accordance with the principle of maximal information entropy formulated by Haken [2].

For $\alpha^{ex} > \alpha_{cr}^{ex}$, the 68 trapped modes form a long-lived background (noise). The information entropy of the relevant part of the system at energy and time scales characteristic of the system is determined by the $K = 2$ fast modes $R_f$, eq. (18). It is almost constant but much smaller than the information entropy below the instability point (Fig. 4). This result is in accordance with the formation of a new (short-lived) order in the system.

The calculated information entropies depend on the basic set of wavefunctions. Therefore, some additional calculations are performed with changed parameters of the Woods-Saxon-potential (Fig. 5) as well as with a larger configuration space (Fig. 6). All the results in Figs. 5 and 6 show the same tendency as those in Figs. 2 and 3: The mixing of the broad modes increases in the instability region much stronger than the mixing of the narrow modes.

In Fig. 7, the $I_\eta$ in relation to the basic set of Slater determinants is drawn. It is $I_\eta(\alpha^{ex} = 0) = I_\eta$ when both calculations are performed with the same $\alpha^{in}$. The value of $I_\eta$ is dominated by the internal mixing the strength of which is constant ($\alpha^{in} = 1$) in our calculations. In spite of the comparably small value of $\alpha^{in}$, the spreading of the shell model states described by (8) is large. Therefore, $I_\eta$ is almost constant as a function of $\alpha^{ex}$ for $\alpha^{ex} < \alpha_{cr}^{ex}$ (Fig. 7). The information entropy of the relevant part of the system at large $\alpha^{ex}$ is significantly smaller than that of the bound system (corresponding to $\alpha^{ex} = 0$). In that sense, the results show, for large $\alpha^{ex}$, qualitatively the same behaviour as those in Fig. 4.
4 Discussion

The behaviour of the information entropy near an instability point is investigated by Haken [2] by using a classical description. According to these results, the information entropy of the unstable modes increases at the instability point much stronger than that of the stable modes. The results obtained by us for the mixing coefficients of the eigenstates of the open nuclear quantum system show a similar behaviour (Figs. 2, 3, 5, 6). They are a signature for the driving role of the fast modes in the process of redistribution at the instability point which, by themselves, are formed under the influence of the environment (decay channels). The trapped modes are "slaved" in the sense that they "follow" the relevant short-lived modes [1].

Additionally, the information entropies calculated by us reflect all the features of the reorganization process which we observed in [1]. At $\alpha^{ex} < 1$, the widths as well as the $I^R_\beta$ of all states are rising with increasing $\alpha^{ex}$. For larger $\alpha^{ex}$ but $\alpha^{ex} < \alpha^{ex}_c$, the widths and the $I^R_\beta$ of a few states increase strongly in comparison with those of the other ones. At $\alpha^{ex} \approx \alpha^{ex}_c$ up to $\alpha^{ex} \approx 4$, the mixing coefficients $I^R_\beta$ of the two relevant modes reach their maximal value and remain more or less constant as a function of $\alpha^{ex}$. In correspondence to this, the two broad modes behave like isolated resonances starting from $\alpha^{ex} \approx 4$ (Fig. 4a in [1]). At $\alpha^{ex} \approx 6$, a new generation of broad modes appears (Fig. 1 in the present paper and Figs. 3 and 4b in [1]). In the same region, we observe a comparably strong increase of the $I^R_\beta$ of some of the trapped modes (Fig. 2).

The maximal degree of mixing (or complexity) $I^R_\beta$ of the wavefunction $\tilde{\Phi}_R$ corresponds to an equal distribution over all basic states $\Phi^{SM}_R$, i.e. $\beta_{R \bar{R}} = \beta$ for all $\beta_{R \bar{R}}$ in (11). In our numerical calculations, the maximal value $I^R_{\beta^{(max)}} = 4.25$ for $N = 70$ states is not reached, neither in Fig. 3 nor in Fig. 5. The difference is surely caused by the correlations between the trapped modes (Fig. 1) which exist at all $\alpha^{ex}$ since every deviation from the statistical independence of the trapped modes leads, necessarily, to $I^R_\beta < I^R_{\beta^{(max)}}$.

Let us now discuss the transfer of information from a certain number of states to other states the number of which is different from the original
one. As an example, the original information contained in a certain state of the Slater determinant, is spread over many shell model states \( R \) with the wavefunctions \( \Phi_R^{SM} \). In this case, the information entropy rises from 0 to the maximal value \( N \cdot \ln N \) if an equal distribution is reached. In the opposite case, if information is transferred from a certain number \( N > 1 \) states to one state, the information entropy is reduced.

Let us define, in our case, a set of functions \( \{ \zeta^c \} \) which carry the influence of each original state \( R \) in relation to the channel \( c \),

\[
\zeta^c_E = \sum_{R=1}^{N} \hat{\kappa}_{cR} \tilde{\Phi}_R ,
\]

represented in relation to the basic set \( \{ \tilde{\Phi}_R \} \) with the coefficients

\[
|\hat{\kappa}_{cR}|^2 = \frac{|\gamma_{Rc}|^2}{\sum_R |\gamma_{Rc}|^2} .
\]

Here, \( |\gamma_{Rc}|^2 \) is the partial width of the state \( R \) in relation to the channel \( c \).

In eq. (19), every state \( R \) is weighted by the degree to which it can be observed in the channel \( c \). Further,

\[
\sum_{R=1}^{N} |\hat{\kappa}_{cR}|^2 = 1 .
\]

For large \( \alpha^c \), one has

\[
\sum_{R=1}^{K} |\hat{\kappa}_{cR}|^2 \approx 1 ,
\]

where \( K \) is the number of open decay channels, since [1]

\[
\sum_{R=K+1}^{N} |\hat{\kappa}_{cR}|^2 \approx 0 .
\]
In analogy to (11), we define

\[ I'^{c} = - \sum_{R=1}^{K} |\tilde{\kappa}_{cR}|^2 \cdot \ln |\tilde{\kappa}_{cR}|^2. \]  

(24)

Then the information entropy of the system is

\[ I' = \sum_{c=1}^{K} I'^{c} \]  

(25)

with the maximal value \( I'^{\text{max}} = K \cdot \ln K \). Comparing this value with \( I^{\text{max}} = N \cdot \ln N \), one gets

\[ I'^{\text{max}} \ll I^{\text{max}}. \]  

(26)

Thus, the information transfer from \( N \) different states to a smaller number \( K \) of states is accompanied by a decrease of the information entropy. Exactly such a situation occurs as a result of the redistribution taking place in the system at \( \alpha^{ex} = \alpha^{ex}_c \). The corresponding decrease of the information entropy can be seen in Figs. 4 and 7.

In (25) and (18), only the relevant degrees of freedom are taken into account while in (17), the sum runs over all states independently of the question whether they are relevant or not. A restriction to the information entropy of the relevant modes is justified in full correspondence with the slaving principle: beyond the instability point, a few modes are relevant at time and energy scales characteristic of the system while the slaved modes represent long-lived noise. The situation is illustrated by means of the poles of the \( S \)-matrix in [11].

In a channel representation of the discrete states, the relevant fast modes have (almost) pure wavefunctions in contrast to the trapped modes the wavefunctions of which are strongly mixed also in this representation. A "channel" is defined here in full analogy to a decay channel: one bound particle moves around a core where the particle, the core as well as the relative motion are described by definite quantum numbers. It is possible, therefore, to define
single-particle quantum numbers in the relevant ordered modes. In the noise, single-particle quantum numbers cannot be specified.

The results shown in Figs. 4 and 7 confirm eq. (26). According to the formation of a new order in the system, the information entropy \( I' \) of the relevant modes at \( \alpha > \alpha_c \) is reduced as compared to the information entropy \( I \) for \( \alpha < \alpha_c \) where all degrees of freedom are equally important. The new order at large \( \alpha \) is created under the influence of the environment of decay channels and reflects the structure of these channels [1].

Thus, the exceeding entropy is not exported into the environment if an ordered state is formed. The reduction of the information entropy takes place inside the system under the influence of the environment by decreasing the effective number of degrees of freedom, i.e. by creating a long-lived noise (background) which takes the main part of the information entropy.

The numerical results obtained by us confirm both the increase of the information entropy as a function of increasing \( \alpha \) up to a certain maximal value due to the formation of noise, and the reduction of the information entropy which accompanies the formation of the relevant short-lived ordered states (eq. (18)).

5 Summary

In the present paper, we have investigated the degree of mixing of the wavefunctions \( \Phi_R \) of an open quantum mechanical system in relation to the basic set of wavefunctions \( \{ \Phi_{SM} \} \) of the corresponding closed system. The Hamiltonian operator of the closed system is hermitean, the eigenvalues and eigenfunctions are real, the Schrödinger equation is linear [1]. The eigenfunctions and eigenvalues are assumed, usually, to contain all the spectroscopic information of the system.

As a result of our investigations, the spectroscopic properties of the open system may differ considerably from those of the closed system. If the coupling to the continuum exceeds a certain critical value, the system reorganizes
in such a manner that the spectroscopic information of the closed system is completely lost. In accordance with this, the wavefunctions $\Phi_R$ differ strongly from the basic wavefunctions $\Phi^{SM}_R$: The degree of complexity $I^R_\beta$ of $\Phi_R$ in relation to the $\{\Phi^{SM}_R\}$ is large. The information entropy $I_\beta$ of the system is increased.

The results obtained numerically by us show two points which appear as a consequence of the redistribution taking place inside the nucleus at the critical coupling strength $\alpha^{Q}_{cr}$:

(i) The information entropy $I_\beta$ of the system increases if all degrees of freedom are taken into account.

(ii) The information entropy $I_\beta$ of the system decreases if one restricts oneself to the relevant degrees of freedom.

That means, at the critical coupling strength $\alpha^{Q}_{cr}$, both order and disorder are created together [10]. The order is represented by a few relevant short-lived modes. The corresponding information entropy is small. The disorder is represented by a long-lived noise (trapped modes). The information entropy of this noise is large. The relations between order and disorder, respectively, and the value of the information entropy are in accordance with the usual accepted relations between these values.

Thus, the evolution of the open quantum mechanical system occurs in agreement with the second law of thermodynamics. In reaching an ordered (relevant) state far from thermal equilibrium, the exceeding entropy is, however, not directly exported into the environment but diminished inside the system by creating (irrelevant) trapped modes (noise) near to thermal equilibrium. The number of relevant degrees of freedom is reduced as a result of the redistribution taking place inside the system at $\alpha^{Q}_{cr}$.

The relevant modes have good single-particle quantum numbers in the sense of channel representation, i.e. one particle moves around a core where the particle, the core as well as the relative motion are described by definite quantum numbers. In the noise, single-particle quantum numbers cannot be specified.

The numerical results showed further that the noise is not structureless.
The trapped modes are correlated with each other. In further investigations, the origin of these correlations will be investigated in detail.

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References


Figure 1

The imaginary part of the complex eigenvalues \( \tilde{E}_R = E_R - \frac{i}{2} \Gamma_R \) of 70 states \( R \) versus \( \alpha^{ex} \). The calculations are performed at \( E = 34.7 \text{ MeV} \) for \( \alpha^{in} = 1, N = 70 \) resonance states and \( K = 2 \) open decay channels. The results are shown in a linear ordinate scale cut at \( \Gamma_R = 1.5 \text{MeV} \).

Figure 2

The mixing coefficients \( I^R_\beta \) of all 70 states \( R \) in relation to the basic set of the shell model wavefunctions \( \Phi^{SM}_R \) versus \( \alpha^{ex} \) (\( \alpha^{in} = 1, E = 34.7 \text{ MeV}, N = 70, K = 2 \)). The \( I^R_\beta \) of the two states with the largest widths are denoted by thick points.

Figure 3

The mixing coefficients \( < I^R_\beta > \) averaged over the two broad modes (f), all 70 modes (all) and the 68 trapped modes (s) in relation to the basic set of the shell model wavefunctions \( \Phi^{SM}_R \) versus \( \alpha^{ex} \) (\( \alpha^{in} = 1, E = 34.7 \text{ MeV}, N = 70, K = 2 \)).

Figure 4

The information entropy \( I_\beta \) for all 70 modes (all) as well as for the relevant (rel) and irrelevant (irr) modes beyond the instability point, in relation to the basic set of the shell model wavefunctions \( \Phi^{SM}_R \) versus \( \alpha^{ex} \) (\( \alpha^{in} = 1, E = 34.7 \text{ MeV}, N = 70, K = 2 \)).

Figure 5

The mixing coefficients \( I^R_\beta \) of all 70 states \( R \) in relation to the basic set of the shell model wavefunctions \( \Phi^{SM}_R \) versus \( \alpha^{ex} \). The Woods-Saxon-potential
is deeper by 10 MeV (5 a) and 20 MeV (5 b), respectively, than in Fig. 2 ($\alpha^{in} = 1, E = 34.7$ MeV, $N = 70, K = 2$). The $I^R$ of the two states with the largest widths in each case are denoted by thick points without regard to a possible exchange in the sequence of the states as a function of $\alpha^{ex}$.

**Figure 6**

The imaginary part of the complex eigenvalues $\tilde{E}_R = \tilde{E}_R - \frac{i}{2}\tilde{R}$ of 190 states $R$ (6 a) and the mixing coefficients $I^R_\beta$ in relation to the basic set of the shell model wavefunctions $\Phi^{SM}_R$ (6 b) versus $\alpha^{ex}$. The configurational space is larger $[(1s)^{-1}(1p)^{-1}(2s,1d)^2]$ than in Fig. 2 $[((1s)^{-1}(1p)^{-1}(2s,1d_{5/2})^2]$ ($\alpha^{in} = 1, E = 34.7$ MeV, $N = 200, K = 2$). The $I^R_\beta$ of the two states with the largest widths in each case are denoted by thick points without regard to a possible exchange in the sequence of the states as a function of $\alpha^{ex}$.

**Figure 7**

The information entropy $I_\eta$ for all 70 modes (all) as well as for the relevant (rel) and irrelevant (irr) modes beyond the instability point, in relation to the basic set of the Slater determinants $\Phi^{(0)}_R$ versus $\alpha^{ex}$ ($\alpha^{in} = 1, E = 34.7$ MeV, $N = 70, K = 2$).
Fig. 4