Maximum Likelihood Parameter Estimation in a GNSS Receiver

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Abstract— **The potential of the SAGE (Space Alternating Generalized Expectation Maximization) algorithm for navigation systems in order to distinguish the line-of-sight signal (LOSS) is to be considered. The SAGE algorithm is a low-complexity generalization of the EM (Expectation-Maximization) algorithm, which iteratively approximates the maximum likelihood estimator (MLE) and has been successfully applied for parameter estimation (relative delay, incident azimuth, incident elevation, Doppler frequency, and complex amplitude of impinging waves) in mobile radio environments. This study discusses a receiver using an antenna array. Whereas we estimate the complex amplitudes, relative delays, Doppler frequencies, and the spatial signature of the impinging waves (incident azimuth and elevation). The results of the performed computer simulations and discussion indicate that the SAGE algorithm has the potential to be a very powerful high resolution method to successfully estimate parameters of impinging waves for navigation systems. SAGE has proven to be a promising method to combat multipath due to its good performance, fast convergence, and low complexity.**

I. Introduction

The quality of the data provided by a Global Navigation Satellite Systems (GNSS) receiver depends largely on the synchronization error with the signal transmitted by the GNSS satellite (navigation signal), that is, on the accuracy in the propagation delay estimation of the direct signal (line-of-sight signal, LOSS). The synchronization of a navigation signal is usually performed by a Delay Lock Loop (DLL), which basically implements an approximation of a maximum likelihood estimator (MLE). The problem which arises is that the order of this estimator (the DLL), which is given by the number of impinging wavefronts is chosen according to the assumption that only the LOSS is present. This means that this estimator tries to estimate the relative propagation delay of only one signal replica. In case the LOSS is corrupted by several superimposed delayed replicas this estimator becomes biased, because of the change of the order of the estimation problem at hand. Thus, multipath can introduce a bias up to a hundred meters of range error when using a standard 1-chip wide DLL [12]. In order to reduce this bias, several methods have been proposed such as the Narrow Correlator [22] or the Double Delta Correlator [10]. Although they achieve much better results than a conventional 1-chip wide DLL in terms of multipath-caused timing bias, their performance is severely degraded in case short delayed multipath is present as for single-antenna receivers their ability to distinguish the LOSS from the reflections is limited by the intrinsic time resolution the signal bandwidth imposes [7]. Relatively short delays are just the case of real-life multipath, where the scatterers are located close to the receiver and the extra path covered by the reflection is shorter than one chip period (about 300 m for a chip rate of 1.023 Mcps). This situation is referred to as coherent multipath.

Thus, in order to perform accurate synchronization in the presence of multipath corrupted signals and especially when coherent multipath is present we follow the approach of obtaining the MLE for estimation problems of higher order. Therefore, signal parameters of a number of superimposed delayed replicas have to be estimated jointly. As this leads to a multi-dimensional non-linear optimization problem the reduction of the complexity of this problem is the most important issue to be solved in order to perform precise positioning in a navigation receiver.

Several techniques have been proposed in the literature to solve the multipath problem in GNSS receivers, (see for instance [23] and [20]). Lately, interesting approaches like [15] and [18] have appeared. The first method applies the maximum likelihood principle to the delay estimation in the presence of multipath and unintentional interference in an antenna array receiver, whereas multipath and interference are modeled as colored noise. The latter develops efficient multipath mitigation techniques (with low-complexity) based on a Newton-type method in single antenna and array antenna navigation receivers. In both works, a connection is made between the multipath estimation problem in navigation systems and the same problem in communication systems; further work of the same authors has appeared more recently in [16], [19], and [6] respectively.

In this paper, in order to cope with the synchronization problem induced by multipath signals in GNSS receivers, we explore the potential application of an iterative method estimating the parameters of all impinging wavefronts in white Gaussian noise which already was assessed for communication systems: the SAGE algorithm (Space Alternating Generalized Expectation Maximization algorithm). However, a detailed study on the capabilities of SAGE for precise time-delay estimation has not been performed yet. The SAGE algorithm [8] is based on the Expectation Maximization (EM) algorithm [4], [11], which facilitates optimizing maximum likelihood cost functions that arise in statistical estimation problems, but with improvements regarding the trade-off between convergence rate and complexity. The potential of the SAGE algorithm in communication systems has been proven in [9] where it is shown that SAGE is a promising candidate for channel estimation in direct-sequence code division multiple access

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systems (DS-CDMA). The intention of this paper is to serve as a study on the application of SAGE for navigation systems based on [2]. The performance of the algorithm is assessed by computer simulations under severe multipath conditions.

II. Data Model

We assume that L narrowband planar wavefronts of wavelength λ, complex amplitude γ_ℓ, delay τ_ℓ, Doppler frequency ν_ℓ, incident azimuth ϕ_ℓ, and incident elevation ϑ_ℓ, $1 \leq \ell \leq L$ are impinging on an array of M, $1 \leq m \leq M$ isotropic sensor elements. The transmission medium is considered linear such that the noise corrupted baseband signal at the antenna output $\mathbf{y}(t) \in \mathbb{C}^{M\times 1}$ can be modelled as a superposition of L wavefronts generated by L point sources and additional complex white Gaussian noise $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$, $\mathcal{N}(0, \sigma_n^2)$. The L point sources are located far from the array such that the direction of propagation is nearly equal at each sensor and the wavefronts are approximately planar (far-field approximation). Thus, the propagation field within the array aperture consists of plane waves and we can write

$$\mathbf{y}(t) = \sum_{\ell=1}^{L} \mathbf{s}_\ell(t) + \mathbf{n}(t), \qquad (1)$$

where $\mathbf{s}_\ell(t)$ is given by

$$\mathbf{s}_\ell(t) = \mathbf{a}_\ell\left(\phi_\ell(t), \vartheta_\ell(t)\right)\ \gamma_\ell(t)\ \mathrm{e}^{\mathrm{j}2\pi\nu_\ell(t)t} c(t-\tau_\ell(t)). \qquad (2)$$

Here, $\mathbf{a}_\ell\left(\phi_\ell(t), \vartheta_\ell(t)\right)$ denotes the steering vector of an uniform rectangular array (URA) of $M = M_x \cdot M_y$ isotropic sensors with M_x elements being displaced by Δ_x along the x-axis and M_y elements displaced by Δ_y along the y-axis as depicted in Fig. 1. Assuming $\Delta_x = \Delta_y = \frac{\lambda}{2}$ (λ is the wavelength corresponding to the carrier frequency), i. e.

$$\mathbf{a}_\ell\left(\phi_\ell(t), \vartheta_\ell(t)\right) = [1, \ldots, \mathrm{e}^{\mathrm{j}\pi\ (m_x\ u_\ell\ +\ m_y\ v_\ell)}, \ldots, \mathrm{e}^{\mathrm{j}\pi\ ((M_x-1)\ u_\ell\ +\ (M_y-1)\ v_\ell)}]^{\mathrm{T}} \qquad (3)$$

$$0 \leq m_x \leq M_x - 1 \ \text{ and } \ 0 \leq m_y \leq M_y - 1 \qquad (4)$$

where

$$u_\ell = \cos\phi_\ell(t)\cos\vartheta_\ell(t) \ \text{ and } \ v_\ell = \sin\phi_\ell(t)\cos\vartheta_\ell(t), \qquad (5)$$

and $m_x \in \mathbb{N}$ and $m_y \in \mathbb{N}$. Fig. 1 gives the definitions of azimuth and elevation for this URA.

In Eq. (2) $c(t-\tau_\ell(t))$ denotes the pseudo-random-noise (PN) sequence with delay $\tau_\ell(t)$. We apply Gold codes [12] as used for the GPS C/A code with code period $T = 1$ ms, 1023 chips per code period each with a time duration $T_c = 977.52$ ns and a rectangular chip shape.

The spatial observations of the URA are collected at N time instances, as $\mathbf{y}[n] = \mathbf{y}(n \cdot T_s)$ with $n = 1, 2, \ldots, N$. The channel parameters are assumed constant during the observation interval $T_N = N \cdot T_s$ since the coherence time

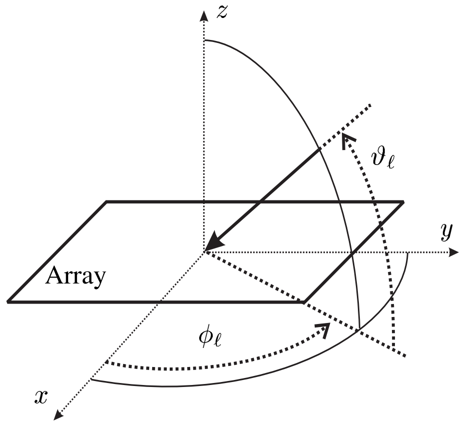

Fig. 1. Definitions of Azimuth ($-180° < \phi_\ell \leq 180°$) and Elevation ($0° \leq \vartheta_\ell \leq 90°$).

of the assumed channel is $T_{coh} \geq T_N$. Collecting the samples of the observation interval leads to

$$\mathbf{Y} = [\mathbf{y}[1], \mathbf{y}[2], \ldots, \mathbf{y}[N]] \in \mathbb{C}^{M\times N}, \qquad (6)$$

$$\mathbf{N} = [\mathbf{n}[1], \mathbf{n}[2], \ldots, \mathbf{n}[N]] \in \mathbb{C}^{M\times N}, \qquad (7)$$

$$\mathbf{S}_\ell = [\mathbf{s}_\ell[1], \mathbf{s}_\ell[2], \ldots, \mathbf{s}_\ell[N]] \in \mathbb{C}^{M\times N}, \qquad (8)$$

$$\boldsymbol{\theta} = \left[\boldsymbol{\theta}_1^{\mathrm{T}}, \boldsymbol{\theta}_2^{\mathrm{T}}, \ldots, \boldsymbol{\theta}_L^{\mathrm{T}}\right]^{\mathrm{T}}, \qquad (9)$$

$$\boldsymbol{\theta}_\ell = [\gamma_\ell, \tau_\ell, \nu_\ell, \phi_\ell, \vartheta_\ell]^{\mathrm{T}}. \qquad (10)$$

Thus, the signal model can be written in matrix notation

$$\mathbf{Y} = \mathbf{S}(\boldsymbol{\theta}) + \mathbf{N} = \sum_{\ell=1}^{L} \mathbf{S}_\ell(\boldsymbol{\theta}_\ell) + \mathbf{N} = \mathbf{A}\,\boldsymbol{\Gamma}\ (\mathbf{C} \odot \mathbf{D}) + \mathbf{N}. \qquad (11)$$

Here, the matrix $\mathbf{S}(\boldsymbol{\theta}) \in \mathbb{C}^{M\times N}$ contains the superimposed impinging wavefronts $\sum_{\ell=1}^{L} \mathbf{S}_\ell(\boldsymbol{\theta}_\ell)$, $\mathbf{A} \in \mathbb{C}^{M\times L}$ denotes the array steering matrix which includes the array steering vector $\mathbf{a}(\phi_\ell, \vartheta_\ell)$ for each impinging wavefront, $\boldsymbol{\Gamma} \in \mathbb{C}^{L\times L}$ is a diagonal matrix which comprises the complex amplitudes of the impinging wavefronts γ_ℓ in its diagonal, $\mathbf{C} \in \mathbb{R}^{L\times N}$ contains the sampled PN sequence for each impinging wavefront $\mathbf{c}(\tau_\ell)$ delayed by τ_ℓ, and $\mathbf{D} \in \mathbb{C}^{L\times N}$ contains the sampled Doppler frequencies for each impinging wavefront

$$\mathbf{d}(\nu_\ell) = \left[\mathrm{e}^{\mathrm{j}2\pi\nu_\ell T_s}, \mathrm{e}^{\mathrm{j}2\pi\nu_\ell\ 2\cdot T_s}, \ldots, \ldots \mathrm{e}^{\mathrm{j}2\pi\nu_\ell\ n\cdot T_s}, \ldots, \mathrm{e}^{\mathrm{j}2\pi\nu_\ell\ N\cdot T_s}\right]^{\mathrm{T}}. \qquad (12)$$

Thus, we get

$$\mathbf{A} = [\mathbf{a}(\phi_1, \vartheta_1)\ \mathbf{a}(\phi_2, \vartheta_2)\ \cdots\ \mathbf{a}(\phi_\ell, \vartheta_\ell)\ \cdots\ \mathbf{a}(\phi_L, \vartheta_L)]\ , \qquad (13)$$

$$\boldsymbol{\Gamma} = \mathrm{diag}\{\gamma_\ell\}_{\ell=1}^{L}, \qquad (14)$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}^{\mathrm{T}}(\tau_1) \\ \mathbf{c}^{\mathrm{T}}(\tau_2) \\ \vdots \\ \mathbf{c}^{\mathrm{T}}(\tau_\ell) \\ \vdots \\ \mathbf{c}^{\mathrm{T}}(\tau_L) \end{bmatrix}, \qquad (15)$$

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}^{\mathrm{T}}(\nu_1) \\ \mathbf{d}^{\mathrm{T}}(\nu_2) \\ \vdots \\ \mathbf{d}^{\mathrm{T}}(\nu_\ell) \\ \vdots \\ \mathbf{d}^{\mathrm{T}}(\nu_L) \end{bmatrix}, \tag{16}$$

whereas $\mathrm{diag}\{\gamma_\ell\}_{\ell=1}^{L}$ represents a diagonal matrix containing the values γ_ℓ, $\ell = 1, 2, \ldots, L$ in its diagonal. In Eq. (11) $\odot$ denotes the Hadamard-Schur product (element-by-element multiplication).

III. Maximum Likelihood Estimation of Superimposed Signals

The problem at hand is to estimate $\boldsymbol{\theta}_\ell = [\gamma_\ell, \tau_\ell, \nu_\ell, \phi_\ell, \vartheta_\ell]^{\mathrm{T}}$, $\ell = 1, 2, \ldots, L$. The estimation of L is not taken care of in this work. In the following we presume L is given.

As we assume spatially and temporally uncorrelated elements in $\mathbf{N}$, the covariance of the noise is $\mathbf{R} = \sigma_n^2 \cdot \mathbf{I}$ and the variance σ_n^2 is considered to be known. Thus, the likelihood function for our signal model is given by the conditional probability density function (pdf)

$$\mathrm{p}(\mathbf{Y}; \boldsymbol{\theta}) = \frac{1}{(\pi\sigma_n^2)^{M \cdot N}} \exp\left(-\frac{\|\mathbf{Y} - \mathbf{S}(\boldsymbol{\theta})\|_{\mathrm{F}}^2}{\sigma_n^2}\right). \tag{17}$$

Here, $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm of a matrix. The maximum likelihood estimator (MLE) is given by

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \{\mathrm{p}(\mathbf{Y}; \boldsymbol{\theta})\}. \tag{18}$$

Estimation of $\boldsymbol{\theta}$ is a computationally prohibitive task since there is no analytical solution for the global maximum, $\mathrm{p}(\mathbf{Y}; \boldsymbol{\theta})$ generally is not a concave function of $\boldsymbol{\theta}$, and $\boldsymbol{\theta}$ and L usually have high dimension. Since the values for maximization of the complex amplitude γ_ℓ can be given in closed form as a function of the other parameters [9], the computation of the MLE for $\boldsymbol{\theta}_\ell = [\gamma_\ell, \tau_\ell, \nu_\ell, \phi_\ell, \vartheta_\ell]^{\mathrm{T}}$ is a $4 \cdot L$-dimensional non-linear optimization procedure.

IV. The SAGE Algorithm

Whereas no closed form expression can be found for the MLE, a numerical approach employs either a grid search or an iterative maximization of the likelihood function. In this work we try an iterative approach, the SAGE algorithm [8], [9], [5] . Instead of performing the above mentioned high dimensional non-linear optimization procedure directly, the SAGE algorithm gives a sequential approximation of the MLE given in Eq. (18) [5] as it performs a sequence of maximization steps in spaces of lower dimension and thereby reducing the complexity considerably. In fact the SAGE algorithm can be considered as a generalization of the well known Expectation-Maximization (EM) algorithm [4], [11]. In the following we sketch the fundamental ideas of the SAGE algorithm.

The basic concept of the SAGE algorithm is the hidden data space [8]. Instead of estimating the parameters of all waves $\boldsymbol{\theta}$ in parallel in one iteration step as done by the EM algorithm, the SAGE algorithm estimates the parameters of each wave $\boldsymbol{\theta}_\ell$ sequentially. Also, instead of estimating the complete $\boldsymbol{\theta}_\ell$, SAGE breaks down the multi-dimensional optimization problem into several smaller problems by conditioning sequentially on a subset of parameters $\boldsymbol{\theta}_S$ while keeping the parameters of the complement subset $\boldsymbol{\theta}_{\bar{S}}$ fixed. We choose the hidden data space as one noisy wave $\mathbf{X}_\ell = \mathbf{S}_\ell + \mathbf{N}_\ell$, where $\mathbf{N}_\ell$ is white Gaussian noise with variance $\beta_\ell\, \sigma_n^2$. This choice of the hidden data space leads to a fast convergence rate and low complexity due to sequential updating and one-dimensional optimization procedures. The stochastic mapping of the hidden data space to the observable signal is $\mathbf{Y} = \mathbf{X}_\ell + \sum_{\substack{\ell'=1 \\ \ell' \neq \ell}}^{L} \mathbf{S}_{\ell'} + \mathbf{N}_{\ell'}$. We choose the sequence of parameter vector $\boldsymbol{\theta}_S(\mu)$ (μ is the parameter update index) as $\boldsymbol{\theta}_S(1) = [\tau_1], \boldsymbol{\theta}_S(2) = [\phi_1], \boldsymbol{\theta}_S(3) = [\vartheta_1], \boldsymbol{\theta}_S(4) = [\nu_1], \boldsymbol{\theta}_S(5) = [\gamma_1], \boldsymbol{\theta}_S(6) = [\tau_2], \boldsymbol{\theta}_S(7) = [\phi_2], \ldots$. For fast convergence we set $\beta_\ell = 1$ [9].

Thus, after estimating the hidden data space in the so-called expectation step (E-step) with

$$\hat{\mathbf{X}}_\ell = \mathbf{Y} - \sum_{\substack{\ell'=1 \\ \ell' \neq \ell}}^{L} \mathbf{S}_{\ell'}(\hat{\boldsymbol{\theta}}_{\ell'}), \tag{19}$$

the so-called maximization step (M-step) with

$$\hat{\tau}_\ell = \arg\max_{\tau_\ell} \left\{ \frac{|\mathbf{a}(\hat{\phi}_\ell, \hat{\vartheta}_\ell)^{\mathrm{H}}\, \hat{\mathbf{X}}_\ell\, (\mathbf{c}(\tau_\ell) \odot \mathbf{d}(\hat{\nu}_\ell))^*|^2}{MN\beta_\ell\sigma_n^2} \right\}, \tag{20}$$

$$\hat{\phi}_\ell = \arg\max_{\phi_\ell} \left\{ \frac{|\mathbf{a}(\phi_\ell, \hat{\vartheta}_\ell)^{\mathrm{H}}\, \hat{\mathbf{X}}_\ell\, (\mathbf{c}(\hat{\tau}_\ell) \odot \mathbf{d}(\hat{\nu}_\ell))^*|^2}{MN\beta_\ell\sigma_n^2} \right\}, \tag{21}$$

$$\hat{\vartheta}_\ell = \arg\max_{\vartheta_\ell} \left\{ \frac{|\mathbf{a}(\hat{\phi}_\ell, \vartheta_\ell)^{\mathrm{H}}\, \hat{\mathbf{X}}_\ell\, (\mathbf{c}(\hat{\tau}_\ell) \odot \mathbf{d}(\hat{\nu}_\ell))^*|^2}{MN\beta_\ell\sigma_n^2} \right\}, \tag{22}$$

$$\hat{\nu}_\ell = \arg\max_{\nu_\ell} \left\{ \frac{|\mathbf{a}(\hat{\phi}_\ell, \hat{\vartheta}_\ell)^{\mathrm{H}}\, \hat{\mathbf{X}}_\ell\, (\mathbf{c}(\hat{\tau}_\ell) \odot \mathbf{d}(\nu_\ell))^*|^2}{MN\beta_\ell\sigma_n^2} \right\}, \tag{23}$$

$$\hat{\gamma}_\ell = \frac{\mathbf{a}(\hat{\phi}_\ell, \hat{\vartheta}_\ell)^{\mathrm{H}}\, \hat{\mathbf{X}}_\ell\, (\mathbf{c}(\hat{\tau}_\ell) \odot \mathbf{d}(\hat{\nu}_\ell))^*}{MN}, \tag{24}$$

is performed. Here, $\mathbf{c}(\tau_\ell)$ denotes the sampled reference PN sequence delayed by τ_ℓ (generated in the receiver). The E-step and the M-step are performed iteratively until the algorithm converges. Eq. (20) is often called rake searcher, as delay estimates $\hat{\tau}_\ell$ correspond to the maximum cross-correlation values. Equivalently, the estimates of azimuth $\hat{\phi}_\ell$ of Eq. (21) and elevation $\hat{\vartheta}_\ell$ of Eq. (22) are given by the maximum spatial correlation. Various choices of the hidden data space could be made which would individually influence the convergence rate. Less informative hidden data spaces yield faster convergence, but more informative hidden data spaces yield an easier M-step [8]. Although the sequence of log-likelihood values form a monotonically non-decreasing sequence of parameter estimates, convergence to even a local maximum is not guaranteed. Therefore, the SAGE algorithm has to be initialized in a region which is close enough to a local (at best the global) maximum. Then the sequence of estimates will converge in norm to it [8].

V. Initialization of the SAGE Algorithm

Several methods are proposed in [9] and [21] in order to perform an initialization of the SAGE algorithm . Since this work should preliminary assess the potential of the SAGE algorithm for navigation systems, we use a technique which commonly is referred to as successive interference cancellation. This method starts with the pre-initial setting $\hat{\boldsymbol{\theta}}_\ell = [0,0,0,0,0]^{\mathrm{T}}$ for each $\ell = 1,2,\ldots,L$. It successively performs a maximum search of correlation processes for the delay and Doppler frequency, and for azimuth and elevation:

$$(\hat{\tau}_\ell, \hat{\nu}_\ell) = \arg\max_{\tau_\ell,\nu_\ell}\left\{|\hat{\mathbf{X}}_\ell\ (\mathbf{c}(\tau_\ell)\odot\mathbf{d}(\nu_\ell))^*|^2\right\}, \quad (25)$$

$$(\hat{\phi}_\ell, \hat{\vartheta}_\ell) = \arg\max_{\phi_\ell\ \vartheta_\ell}\left\{|\mathbf{a}(\phi_\ell,\vartheta_\ell)^{\mathrm{H}}\hat{\mathbf{X}}_\ell\ (\mathbf{c}(\hat{\tau}_\ell)\odot\mathbf{d}(\hat{\nu}_\ell))^*|^2\right\}. \quad (26)$$

In order to perform the E-step, it subtracts signal estimates of waves whose parameter estimates already have been initialized based on the observed data $\mathbf{Y}$. Since in Eq. (25) ϕ_ℓ and ϑ_ℓ are unknown, $\hat{\mathbf{X}}_\ell$ is summed incoherently over all sensors to provide the initial delay and Doppler estimates. The estimates of the complex amplitudes γ_ℓ can be derived from Eq. 24.

In order to ensure fast convergence and good estimation results, the initialization of the SAGE algorithm in a GNSS receiver could be augmented by appropriate *a priori* knowledge of the parameters of the impinging waves. For example, we could consider the estimates of the delay and Doppler frequency provided through acquisition and additionally one could use almanac data to get initial estimates for the spatial signature of the LOSS. Furthermore, a good initialization or *a priori* knowledge of the parameters which are to be estimated will also reduce the search space of the parameters $\boldsymbol{\theta}$ and therefore reduce complexity of the optimization procedures.

We assume that there is no knowledge of almanac data and only rough acquisition estimates available which indicate the delay of the LOSS with an accuracy of two chips. This facilitates the initialization procedure considerably.

VI. Simulation Results

We tested the SAGE algorithm under severe multipath conditions via Monte-Carlo simulations. A receiver using an array antenna with M antenna elements is considered. An URA is simulated with $M = M_x \cdot M_y$, and $\Delta_x = \Delta_y = \frac{\lambda}{2}$. The channel parameters are assumed constant during the observation interval T_N, the one-sided bandwidth is B =1.023 MHz and the sampling frequency is $f_s = 1/T_s = 2 \cdot B$. We apply the initialization as discussed in Section V. In order to describe the behavior and to assess the performance of the SAGE algorithm we adopt the root mean square error (RMSE) and the Cramer-Rao lower bound (CRLB). The number of iteration cycles performed by the SAGE algorithm is denoted by $k \in \mathbb{N}$ and the mean number of iteration cycles until convergence of the algorithm is given by $\bar{k} \in \mathbb{R}$.

Signal-to-noise ratio (SNR) denotes the LOSS-to-noise ratio. The effective SNR in dB can be obtained following [3] as

$$\mathrm{SNR} = \mathrm{C/N_0} - 10 \cdot \log_{10}(2 \cdot B) + 10 \cdot \log_{10}(N_c), \quad (27)$$

whereas $\mathrm{C/N_0}$ in dB-Hz denotes the carrier-to-noise density ratio and $N_c \in \mathbb{N}$ is the number of code periods within T_N. Here, SNR means pre-correlation SNR but we take into account the observation time T_N given by the number of code periods N_c for which correlation is averaged. Typical values of $\mathrm{C/N_0}$ for GPS range from 35 - 55 dB-Hz. Please notice that $\mathrm{C/N_0}$ is dependent on the receiver and system noise figure, noise temperature, and on elevation of the satellite. Following [14], [13] an aviation navigation receiver being interoperable with a standard GPS antenna without preamplifier, shall be capable of tracking GPS satellites with a minimum input signal power of - 166 dBW at the receiver port in the presence of background thermal noise density of -206.6 dBW/Hz. This leads to a minimum $\mathrm{C/N_0}$ of 40.3 dB-Hz for a GPS aviation receiver. In order to achieve SNR ranging from -15 dB to -5 dB for $\mathrm{C/N_0} = 40.3$ dB-Hz and $B = 1.023$ MHz, the observation time T_N is ranging from 5 ms to 50 ms .

In order to assess the performance of the SAGE algorithm with respect to delay estimation of the LOSS for a GNSS receiver, we analyze the behavior of our estimator for a single reflective multipath as a function of its relative delay to the LOSS ($L = 2$). This procedure is commonly used in GNSS literature in order to characterize multipath mitigation capabilities. In the following parameters with the subscript 1 refer to the LOSS and parameters with the subscript 2 refer to the reflection. Thus, the signal parameters of the LOSS are $\theta_1 = [\gamma_1, \tau_1, \nu_1, \phi_1, \vartheta_1]^{\mathrm{T}}$ and the signal parameters of the multipath are $\theta_2 = [\gamma_2, \tau_2, \nu_2, \phi_2, \vartheta_2]^{\mathrm{T}}$. The reflected multipath and the LOSS are considered to be in-phase $\arg\gamma_1 = \arg\gamma_2$, which corresponds to the worst possible case [20], [15], [18]. The signal-to-multipath ratio (SMR) is 5 dB for all reflections (cf. [20], [16], [15]). The parameter estimates are quantized to a precision of 0.5 ns for $\hat{\tau}_\ell$, 0.5 Hz for $\hat{\nu}_\ell$, and $0.1°$ for $\hat{\phi}_\ell$ and $\hat{\vartheta}_\ell$. Convergence of the SAGE algorithm is reached if for all impinging waves $\ell = 1,2,\ldots,L$ all the following constraints are fulfilled: $|\hat{\tau}_\ell^{(k)} - \hat{\tau}_\ell^{(k-1)}| \leq 0.5$ ns, $|\hat{\nu}_\ell^{(k)} - \hat{\nu}_\ell^{(k-1)}| \leq 0.5$ Hz, $|\hat{\phi}_\ell^{(k)} - \hat{\phi}_\ell^{(k-1)}| \leq 0.1°$, $|\hat{\vartheta}_\ell^{(k)} - \hat{\vartheta}_\ell^{(k-1)}| \leq 0.1°$, $||\hat{\gamma}_\ell^{(k)}| - |\hat{\gamma}_\ell^{(k-1)}|| \leq 0.01$, $|\arg\hat{\gamma}_\ell^{(k)} - \arg\hat{\gamma}_\ell^{(k-1)}| \leq 0.01$.

In the following we will assess the performance of a GNSS receiver using an URA with $M = 9$. Fig. 2, Fig. 3, Fig. 4, and Fig. 5 demonstrate the estimation performance of the SAGE algorithm for the delay $\hat{\tau}_1$, the Doppler $\hat{\nu}_1$, the azimuth $\hat{\phi}_1$, and the elevation $\hat{\vartheta}_1$ of the LOSS for a scenario with $\phi_1 = 50°$, $\phi_2 = 102°$, $\vartheta_1 = 40°$, $\vartheta_2 = 26°$, and absolute relative Doppler difference $\Delta\nu = |\nu_1 - \nu_2| \cdot T_N = 0.5$.

Fig. 2 illustrates that the proposed estimator attains the CRLB for spatially non-coherent wavefronts. Compared to the MLE and the hybrid beamformer presented in [15], [16] and the proposed subspace methods presented in [17], [1], the SAGE algorithm is able to precisely estimate the delay of the LOSS even if multipath with small relative delay $\Delta\tau = |\tau_1 - \tau_2|/T_c$ and small absolute relative Doppler difference $\Delta\nu$ is present. This is mainly due to the fact that the MLE which is given in Eq. (18) additionally estimates the complex amplitudes γ_ℓ, the Doppler frequencies ν_ℓ, the azimuths ϕ_ℓ, and the elevations ϑ_ℓ of all impinging waves explicitly, which is exploited to enhance the resolution abilities especially for small $\Delta\tau$ and small $\Delta\nu$, but increases the complexity of the estimator. If the complex amplitudes γ_ℓ, the Doppler frequencies ν_ℓ, the azimuths ϕ_ℓ, and the elevations ϑ_ℓ are not explicitly estimated the CRLB for $\hat{\tau}_1$ of such a MLE will tend to infinity towards small relative delays $\Delta\tau$ and small $\Delta\nu$. This is the case for the MLE given in [15], [16]. Thus, the MLE as given in Eq. (18) is very much suitable for the estimation of synchronization parameters for navigation applications and can be solved with reasonable complexity by the SAGE algorithm. The results have shown that SAGE can be considered as a high resolution method for precise synchronization.

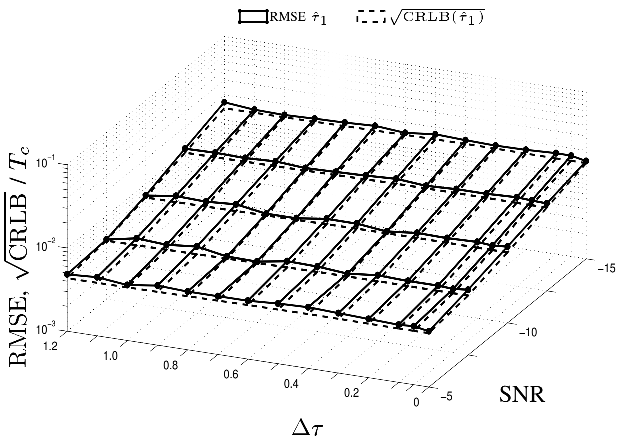

Fig. 2. RMSE of the time-delay of the LOSS / T_c versus the relative delay $\Delta\tau$ and the SNR. Parameters: $\phi_1 = 50°$, $\phi_2 = 102°$, $\vartheta_1 = 40°$, $\vartheta_2 = 26°$, $L = 2$, $M = 9$, $\Delta\nu = 0.5$.

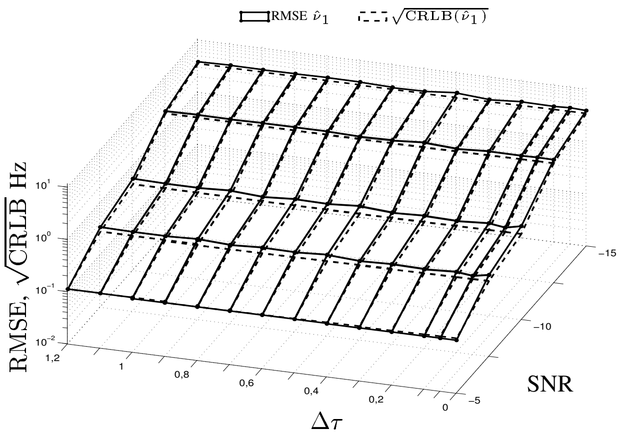

Fig. 3. RMSE of the Doppler frequency of the LOSS in Hz versus the relative delay $\Delta\tau$ and the SNR. Parameters: $\phi_1 = 50°$, $\phi_2 = 102°$, $\vartheta_1 = 40°$, $\vartheta_2 = 26°$, $L = 2$, $M = 9$, $\Delta\nu = 0.5$.

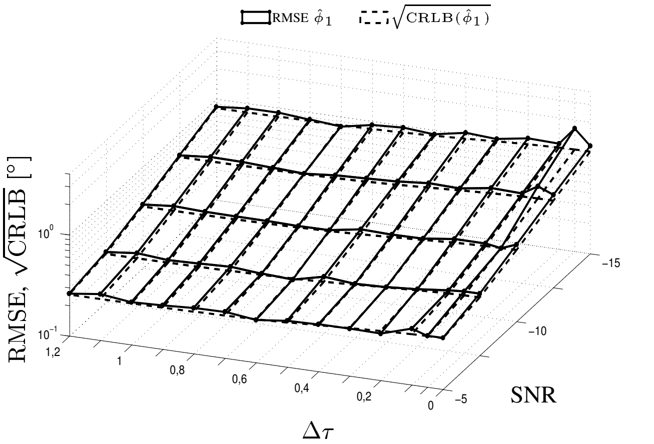

Fig. 4. RMSE of the the azimuth of the LOSS in degrees versus the relative delay $\Delta\tau$ and the SNR. Parameters: $\phi_1 = 50°$, $\phi_2 = 102°$, $\vartheta_1 = 40°$, $\vartheta_2 = 26°$, $L = 2$, $M = 9$, $\Delta\nu = 0.5$.

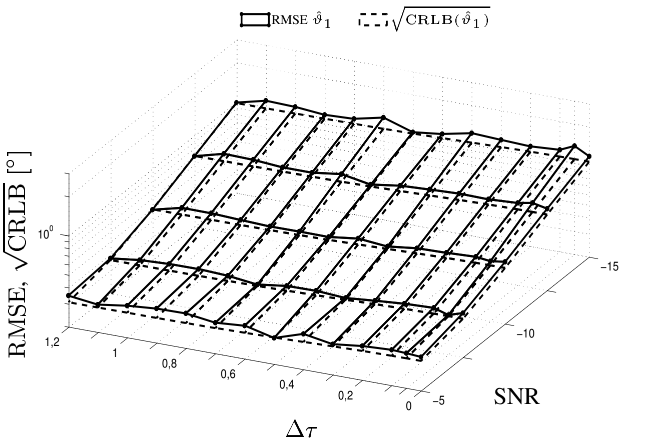

Fig. 5. RMSE of the elevation of the LOSS in degrees versus the relative delay $\Delta\tau$ and the SNR. Parameters: $\phi_1 = 50°$, $\phi_2 = 102°$, $\vartheta_1 = 40°$, $\vartheta_2 = 26°$, $L = 2$, $M = 9$, $\Delta\nu = 0.5$.

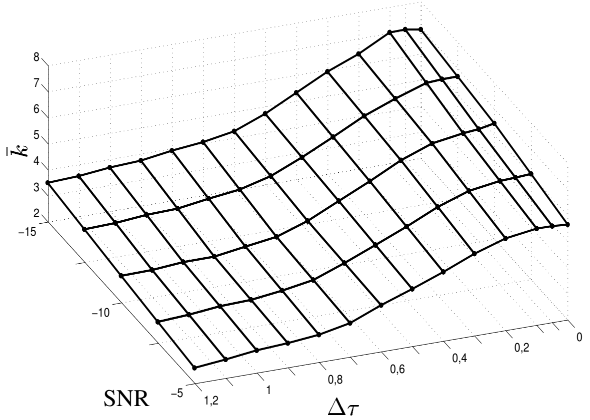

Fig. 6. Mean number of iteration cycles $\bar{k}$ versus the relative delay $\Delta\tau$ and the SNR. Parameters: $\phi_1 = 50°$, $\phi_2 = 102°$, $\vartheta_1 = 40°$, $\vartheta_2 = 26°$, $L = 2$, $M = 9$, $\Delta\nu = 0.5$.

VII. CONCLUSION

In this work we have addressed the problem of estimating the propagation time-delay of the LOSS in a GNSS receiver under the presence of severe multipath (SMR = 5 dB). We tested the SAGE algorithm in order to reduce the complexity in solving the MLE given in Eq. (18). The SAGE algorithm iteratively approximates the MLE and significantly reduces the complexity by breaking down the multi-dimensional non-linear optimization problem of the MLE as given in Eq. (18) into a number of one-dimensional ones.

For an URA with $M = 9$ SAGE is capable of precisely estimating the delay of the LOSS under presence of severe reflective multipath, even if the relative time-delay $\Delta\tau$ and the absolute relative Doppler difference $\Delta\nu$ is small.

Thus, SAGE has proven to be a very promising method to combat multipath for navigation applications, especially for safety critical applications like aviation due to its good performance and low complexity.

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