

On the Differential Propagation Phase in Polarimetric Weather Radar Measurements

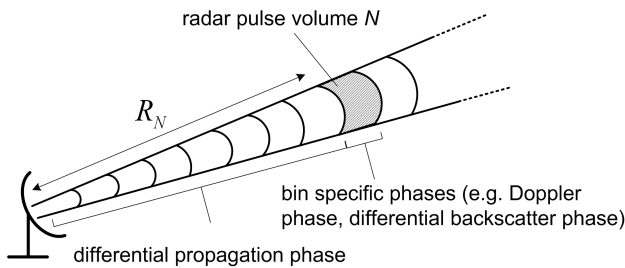
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Abstract— Besides differential attenuation the differential phase due to propagation in anisotropic media like precipitation is one of the mechanisms that is responsible for depolarisation of polarised electromagnetic waves in the microwave region. Modern coherent polarimetric weather radars are able to measure the differential propagation phase and use it as additional parameter for weather remote sensing. A review on the historical development of the use of differential propagation phase in weather radar technology will be given, including a critical assessment and overview of up-to-date applications.

I. INTRODUCTION

For weather services all over the world weather radars are the key instruments for the rain rate retrieval over large areas. They provide besides satellite measurements the possibility to track the development and the evolution of clouds and storms. For now-casting weather radars are the most valuable tool to measure the rain rate.



$$\Phi_{dp}^{2-way} = 2 \cdot \frac{180}{\pi} \cdot \Re(K_h - K_v) \cdot R_N$$

Fig. 1. Geometry of a monostatic weather radar.

Nowadays operational weather radars usually process only the amplitude of the backscattered signal and the phase if they have Doppler capabilities at one polarisation. After extensive research efforts in the last decades polarimetric weather radars become now state-of-the-art and operational also in Germany within the next years.

The use of polarisation provides additional information due to the anisotropy of precipitation. Ice crystals in the top of the clouds have the form of needles or plates, e.g. Aydin and Tang (1997), and are oriented horizontally as one can

observe the sub-sun effect from airplanes, Katz (1998) and references herein. Raindrops have an oblate shape whereas their oblateness depends on their size, e.g. Pruppacher and Pitter (1971).

For free-space propagation one can solve Maxwell's equations to the well known Helmholtz equations for the electric and magnetic field. The solution of the electric field for a plane TEM - wave is given by

$$\vec{E}(z, t) = \vec{E}_0 e^{-jkz} \cdot e^{j\omega t} \quad (1)$$

where \vec{E}_0 is the complex vector amplitude of the electric field, z the propagation direction, $j = \sqrt{-1}$, ω the angular frequency, t the time and the complex propagation constant k is

$$k = \beta - j\alpha \quad (2)$$

with the phase constant β in $rad \cdot m^{-1}$ and the attenuation constant α in $Np \cdot m^{-1}$. For anisotropic media as precipitation the complex propagation constant shows a different behaviour for different propagation directions and also for different polarisations where the polarisation dependent propagation constants at horizontal and vertical polarisation are given by

$$K_{h,v} = k_0 + k_{h,v} \quad (3)$$

where k_0 is the free-space propagation constant in m^{-1} and $k_{h,v}$ are the polarisation dependent complex propagation constants for horizontal and vertical polarisations respectively.

For precipitation the complex propagation constants are given by van de Hulst (1957)

$$K_{h,v} = k_0 + \frac{2\pi}{k_0} \int_D f_{hh,vv}(D) \cdot N(D) dD \quad (4)$$

where $f_{hh,vv}$ are the complex forward scattering amplitudes in mm and $N(D)$ is the drop-size distribution in $mm^{-1}m^{-3}$.

The one-way differential propagation phase in degrees is given by the phase difference of an horizontal and vertical polarised wave traveling along a propagation path L in km

$$\phi_{dp}^{1-way} = \frac{180}{\pi} \cdot \Re(K_h - K_v) \cdot L. \quad (5)$$

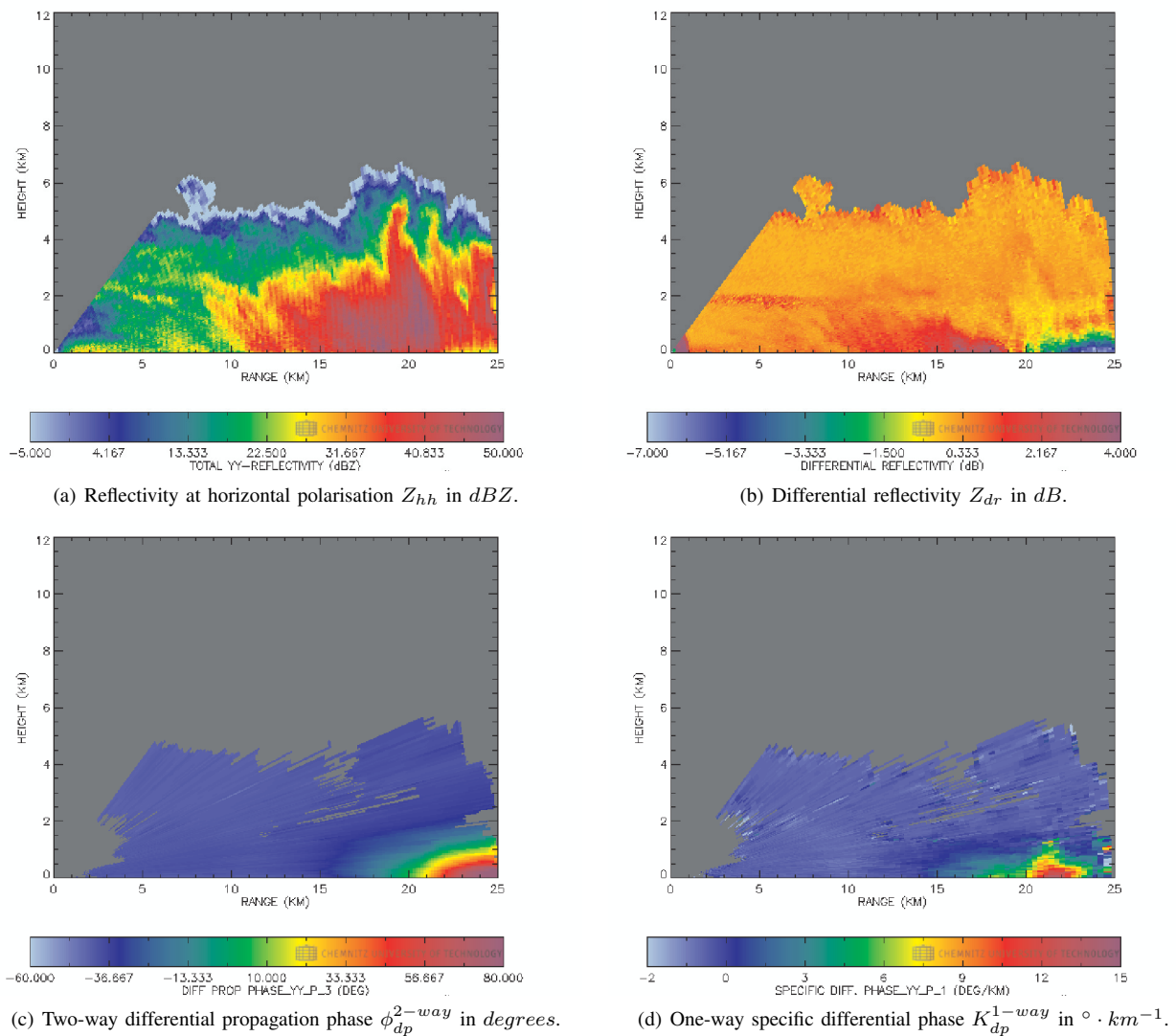


Fig. 2. Range-height indicators (RHI's) of DLRs POLDIRAD of a convective summer storm.

As one can see the differential propagation phase is a range cumulative quantity that becomes larger the longer the propagation path is. For heavy precipitation and long propagation paths phase wrapping may occur.

Fig. 1 shows the radar geometry of a monostatic weather radar. The radar transmits pulses of typically $1 \mu s$ with the pulse repetition frequency PRF . The PRF determines the maximum available range, whereas the pulse width determines the resolution of the radar pulse volumes along the range.

II. HISTORIC OUTLINE OF RESEARCH EFFORT

A. The 1970s - The first appearance of ϕ_{dp}

In the early 1970s the differential propagation phase was first considered by communication engineers as one disturbing factor creating cross-talk in microwave links. To get an idea of the impact of the differential propagation phase on microwave links scattering simulations were performed e.g. by Watson

and Arabi (1973), Morrison et al. (1973) and Oguchi and Hosoya (1974).

McCormick and Hendry (1975) did a pioneering work in the field of polarimetric weather radars at the National Research Council of Canada. They were the first ones that reported measurements of the differential propagation phase at S-band (10.4 cm wavelength) and Ku-band (1.8 cm wavelength) with polarimetric weather radars operating at circular polarisation basis, Hendry et al. (1976).

B. The 1980s - ϕ_{dp} on its way into the heads of the radar meteorologists

Inspired by the work of Seliga and Bringi (1976) and Seliga and Bringi (1978) the weather radar community started to use the polarisation basis horizontal and vertical in the 1980s. Alternately horizontal and vertical polarised pulses are transmitted and the backscattered echoes are received in either horizontal or vertical polarisation basis or simultaneously with

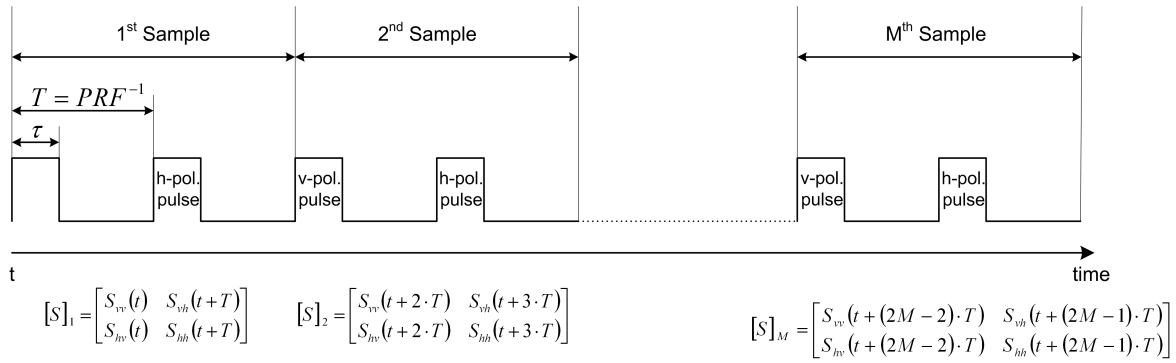


Fig. 3. Measurement scheme of weather radars transmitting alternately horizontal and vertical polarised pulses and receiving the backscattered echoes simultaneously in both polarisation channels.

a two-channel receiver.

Examples of radar data at horizontal/vertical polarisation basis is shown in figs. 2. Shown is a vertical cut through a convective storm measured by the monostatic C-band Polarisation Diversity Radar POLDIRAD (5.45 cm wavelength) of the German Aerospace Center (DLR), Oberpfaffenhofen. A technical description of the radar is given by Schroth et al. (1988).

All weather radars are capable to measure the reflectivity as shown in fig. 2 (a). The reflectivity is proportional to the received power. The connection of the reflectivity with the microphysical properties of precipitation is given by

$$z_{hh} = \frac{\lambda^4}{\pi^5 |K|^2} \int_D \sigma_{hh}(D) N(D) dD \quad (6)$$

with the wavelength λ in mm, $|K|^2$ is the dielectric constant of the precipitation and σ_{hh} is the backscattering cross-section in mm^2 of a single raindrop with the equivalent diameter D . Because the dynamic range of the reflectivity is large it is usually given in dBZ. Where 0 dBZ correspond to the reflectivity of one raindrop with the diameter of 1 mm in a one cubic meter volume.

$$Z_{hh} = 10 \cdot \log_{10} \left(\frac{z_{hh}}{mm^6 \cdot m^{-3}} \right) \quad (7)$$

The reflectivity is determined via the radar equation for distributed targets by measuring the backscattered power as

$$Z_{hh} = 10 \cdot \log_{10} (C \cdot P_{hh} \cdot R_N^2) \quad (8)$$

where C is the radar constant, P_{hh} is the power received in the horizontal polarised channel when a horizontal polarised pulse was transmitted and R_N is the range between the radar and the pulse volume as shown in fig. 1.

Polarimetric weather radars are capable to measure also the anisotropy of precipitation permitting hydrometeor classification and hence a deeper insight into the cloud structure. The differential reflectivity shown in fig. 2 (b) is a measurable of anisotropy related to the backscattering properties of the particles

$$Z_{dr} = 10 \cdot \log_{10} \left(\frac{\int_D \sigma_{hh} N(D) dD}{\int_D \sigma_{vv} N(D) dD} \right). \quad (9)$$

Z_{dr} could be determined by incoherent polarimetric radars measuring the power in both the horizontal and vertical polarised channel and is usually given in dB as

$$Z_{dr} = 10 \cdot \log_{10} \left(\frac{P_{hh}}{P_{vv}} \right). \quad (10)$$

For weather returns measured by ground based weather radars the differential reflectivity usually has positive dB values because almost all hydrometeors are aligned with their major axis in the horizontal plane with the result of higher backscattering cross-sections at horizontal polarisation than at vertical polarisation for microwave frequencies.

The zone of negative Z_{dr} values in fig. 2 (b) at the range of more than 20 km away from the radar is due to strong differential attenuation. Which means that the power in the horizontal polarised channel is much stronger attenuated than the power in the vertical polarised channel due to the oblateness of the hydrometeors.

Coherent radars with polarisation diversity as the POLDIRAD are able to measure also the two-way differential propagation phase ϕ_{dp}^{2-way} as shown in fig. 2 (c). One can see clearly the range cumulative character and one of the main advantage that the differential propagation phase is unaffected by attenuation effects as long as a backscattered signal is measurable and not below the detection level of the radar.

To show the calculation of the differential propagation phase from measurements of alternate transmitting horizontal and vertical polarisation radars first their measurement principle will be explained more in detail, fig. 3.

The differential propagation phase is the phase difference between the horizontal and the vertical polarised pulse, i.e. $\arg \langle S_{hh} S_{vv}^* \rangle$ where $\langle \rangle$ denotes the average over M samples. But this phase also contains a Doppler phase shift because between the measurement of the two S-matrix elements in alternate horizontal/vertical mode there is the time delay of the pulse repetition time T which is dependent on the pulse repetition frequency PRF . This fact was first pointed out

by Chandra et al. (1984). They suggested to simply subtract the Doppler phase which is measured between two equally polarised pulses as

$$\phi_{dp,1}^{2-way} = \frac{\arg \langle S_{hh}(t+T)S_{vv}^*(t) \rangle - \arg \langle S_{vv}^*(t)S_{vv}(t+2T) \rangle}{2} \quad (11)$$

A second possibility was introduced by Jameson and Mueller (1985)

$$\phi_{dp,2}^{2-way} = \frac{\arg \langle S_{vv}^*(t)S_{hh}(t+T) \rangle + \arg \langle S_{hh}(t+T)S_{vv}^*(t+2T) \rangle}{2} \quad (12)$$

The statistics of these formulae were treated in detail by Sachidananda and Zrnic (1986). They showed that the standard deviation of the differential propagation phase is dependent on the sample size M , the Doppler spectrum width and the correlation between the two co-polarised S-matrix elements $|\rho_{hv}|$, eqn. (13). It is obvious that eqn. (12) yields to more accurate results due to the fact that only one lag parameters are used which are higher correlated to each other.

$$|\rho_{hv}| = \frac{|\langle S_{hh}S_{vv}^* \rangle|}{\sqrt{(\langle |S_{hh}|^2 \rangle \langle |S_{vv}|^2 \rangle)}} \quad (13)$$

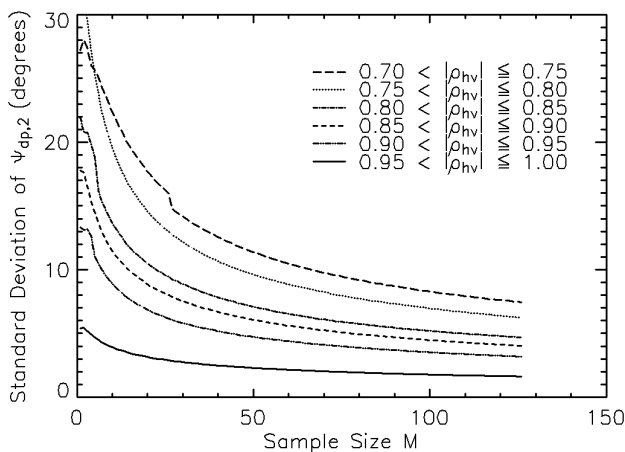


Fig. 4. Standard deviation of differential propagation phase $\phi_{dp,2}^{2-way}$ in dependence of the sample size M and the magnitude of the co-polar correlation coefficient $|\rho_{hv}|$.

Calculations concerning the standard deviation of the differential propagation phase at C-band were performed by Hubbert et al. (1993). They could be proved with real data of the POLDIRAD by Otto and Chandra (2007), fig. 4, showing that the differential propagation phase could be determined with an accuracy from 1 up to 2 degrees for $|\rho_{hv}|$ greater than 0.95.

C. The 1990s until nowadays - ϕ_{dp} becomes operational

In the 1990s a lot of work was done, algorithms created to use ϕ_{dp} also in operational weather radars. Some of that work is summarised in the next chapters.

Since a couple of years operational polarimetric weather radars are installed worldwide. Because of the complexity and the costs of a high power polarisation switch these do not use the two-shot method transmitting alternately horizontal and vertical polarised pulses. However the trend seems to be to transmit a horizontal and vertical pulse at once and receive the echo simultaneously in a horizontal and a vertical polarised channel as suggested by Sachidananda and Zrnic (1985).

At this polarisation basis the differential propagation phase can be instantaneously estimated as the phase difference between the horizontal and vertical polarised receiver channel, no Doppler correction is necessary. Because of that the standard deviation of the differential propagation phase is no longer dependent on the Doppler spectrum width but due to cross-talk between the receiver channels and canted hydrometeors the co-polar correlation coefficient may be less than for the two-shot method. A big issue for operational weather radars is also the need for fast scanning times of the weather around the radar permitting not the time to take 64 or 128 samples per radar pulse volume which strongly affects the quality of all estimates.

III. PROS AND CONS OF ϕ_{dp}

Although the differential propagation phase has a lot of drawbacks there are advantages that make it essential to use it in upcoming operational radars.

The most important references which provide discussions on the pros and cons of the differential propagation phase are Sachidananda and Zrnic (1986), Blackman and Illingworth (1995) and Zrnic and Ryzhkov (1996):

- the differential propagation phase is independent of the radar transmitter and receiver calibration, its measurement is self-calibrating,
- it is the only measurement quantity of monostatic weather radars that is dependent on the forward scattering properties of the precipitation medium,
- it is not affected by signal attenuation and may be a valuable tool to measure rain rate even at low elevation angles with partially blocked beams as long the signal does not drop below the detection level,
- in comparison to reflectivity it is less sensitive to variations of the drop-size distribution,
- it should not be affected by tumbling hail in rain-hail mixtures permitting rain rate retrieval even in these cases.

The biggest advantages are the independence of signal attenuation and the radar calibration. In operational polarimetric weather radars these two properties will be the main push to improve the data quality.

The drawbacks of the differential propagation phase are:

- it is sensitive to ground clutter because ground clutter introduces a random differential phase shift upon backscat-

ter that can not be distinguished from the differential forward scattering effect,

- it is sensitive to any differential phase shift upon backscatter which becomes a more challenging problem at higher frequencies as C-band (5 cm wavelength) and X-band (3 cm wavelength),
- it could be affected by reflectivity gradients within the radar pulse volume, cf. appendix of Ryzhkov and Zrnice (1996),
- rain rate estimates based on differential propagation phase measurements have a low resolution because of the path-integrated nature of the differential propagation phase in comparison to reflectivity based estimators,
- it is noisy at low rain rates.

The most challenging problem of these is probably the differential backscatter phase δ_{co} upon backscattering that occurs when it comes to non-Rayleigh scattering. So it is mainly a problem at C- and X-band frequencies where the electrical size of the hydrometeors becomes a larger fraction of the wavelength. There are indications that δ_{co} may be a more severe problem at C- than X-band as pointed out by Ryzhkov and Zrnice (2005) due to a resonance effect that was found by Holt and Evans (1977) after performing scattering simulations.

IV. SPECIFIC DIFFERENTIAL PHASE K_{dp}

In algorithms the range derivative of the differential propagation phase is used very often, the one-way specific differential phase K_{dp}^{1-way} in $^{\circ} \cdot km^{-1}$, eqn. (14), where $r_1 < r_2$ are two ranges along the radar ray. In fig. 2 (d) the specific differential phase is shown. In the region behind 20 km where the differential reflectivity is severely affected due to differential attenuation the specific differential phase still gives a good impression of the precipitation strength.

$$K_{dp}^{1-way} = \frac{\phi_{dp}^{2-way}(r_2) - \phi_{dp}^{2-way}(r_1)}{2 \cdot (r_2 - r_1)} \quad (14)$$

Usually the processing scheme for the specific differential phase is as follows:

- unwrapping of the differential propagation phase,
- threshold to check the quality of the data, e.g. with the co-polar correlation coefficient $|\rho_{hv}|$,
- low pass filter of the differential propagation phase profile, may include a heavy filtering for strong weather events to suppress differential backscatter phase influences,
- calculation of K_{dp}^{1-way} .

Two algorithms to process K_{dp}^{1-way} are presented in detail and compared in Brandes et al. (2001).

The trend in the newest publications and operational applications is to avoid the calculation of the specific differential phase and use the differential propagation phase directly as input for the algorithms. This is because the difficulties that arise in the estimation of K_{dp}^{1-way} due to the drawbacks mentioned, namely the differential backscatter phase and non-uniform beam-filling.

V. APPLICATIONS OF DIFFERENTIAL PROPAGATION PHASE

A. Rain rate estimation with K_{dp}

To estimate the rain rate from differential propagation phase measurements the most simplest method are $R - K_{dp}$ relationships with the rain rate R in $mm \cdot h^{-1}$ which are found by scattering simulations assuming specific drop-size distributions, drop shape models and other environmental conditions as temperature. They have usually the form $R = a \cdot K_{dp}^b$ where b is a coefficient close to one.

A lot of $R - K_{dp}$ relations at different frequencies are available in literature. It is obvious that depending on the inputs to the scattering simulation the $R - K_{dp}$ relationships spans over a large range creating errors in the rain rate estimation. This problem is the same for reflectivity based methods because the drop-size distribution within precipitation is unknown and varies spatially and temporally.

Figs. 5 show the span of some $R - K_{dp}$ relationships for S-band and C-band frequencies. The plotted relationships for S-band were taken from Sachidananda and Zrnice (1987), Jameson (1991), Bringi and Chandrasekar (2001) and Illingworth (2003). The relations for C-band were taken from Jameson (1991), Aydin and Giridhar (1992), Scarchilli et al. (1993), Keenan et al. (2001) and Illingworth (2003).

Tan (1991) made extensive calculations with different drop shape models, different drop-size distributions and different temperatures to explore the variability of $R - K_{dp}$ relationships.

To account for the large variability in $R - K_{dp}$ relations more complex expressions were developed to measure the rain rate. A successful way seems to be the inclusion of the differential reflectivity Z_{dr} in a $R(K_{dp}^{1-way}, Z_{dr})$ relationship. Examples of those for different frequencies could be found in Bringi and Chandrasekar (2001).

B. Correction for attenuation

The origin of attenuation correction for weather radars was provided by Hitschfeld and Bordan (1954). They developed the notation for iterative attenuation correction along one radar ray using attenuation - reflectivity relationships. In this paper there is already stated that this iterative algorithm is instable due to even small calibration errors of the radar. In the worst case the error due to the correction could be larger than the error due to attenuation itself.

Because the differential propagation phase is unaffected by signal attenuation it provides an excellent basis for attenuation correction of measured reflectivities and differential reflectivity.

In fact a close relation between the differential propagation phase shift and the attenuation shows up. Bringi et al. (1990) gave empirical relationships between the specific attenuation at horizontal polarisation A_h , differential attenuation $A_{hv} = A_h(dB \cdot km^{-1}) - A_v(dB \cdot km^{-1})$ and the specific differential phase K_{dp} at S-, C- and X-band frequencies. But for those relationships there is the same misery as for $R - K_{dp}$ relationships namely the dependence on the drop-shapes and

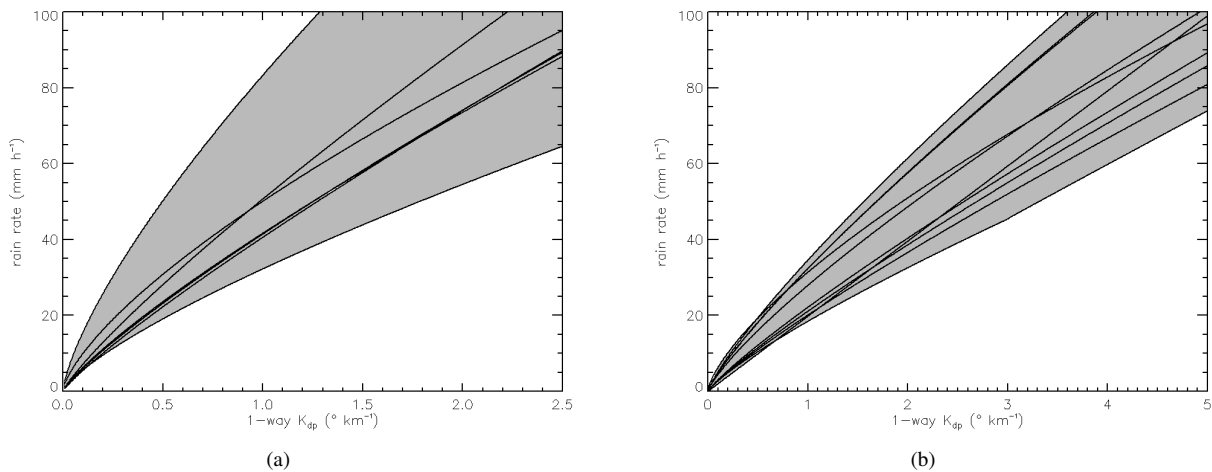


Fig. 5. $R - K_{dp}^{1-way}$ relationships, (a) S-band (10 cm wavelength), (b) C-band (5 cm wavelength).

drop-size distributions. The $A_h - K_{dp}$ relation is also strongly dependent on the temperature because so is the imaginary part of the refractive index of water. Carey et al. (2000) showed the variability of these relations available in literature at C-band.

The easiest way would be to iteratively determine the attenuation using an $A_h - K_{dp}$ or $A_{hv} - K_{dp}$ relationship along the ray to correct for attenuation. But also this iterative attenuation correction will not be accurate and numerically unstable as the algorithm by Hitschfeld and Bordan (1954).

To stabilise iterative correction algorithms Meneghini et al. (1983) extend the formulation of Hitschfeld and Bordan (1954) including the path-integrated attenuation (PIA) as constraint. In their formulation the specific attenuation in $dB \cdot km^{-1}$ at one range along the ray is given as function of the attenuated reflectivity at this point, the PIA and the attenuated reflectivity at the range where the PIA was determined.

To estimate the PIA with weather radars different approaches appeared in literature. Delrieu et al. (1997) suggested mountain returns and Smyth and Illingworth (1998) the use of negative values of differential reflectivity Z_{dr} behind a strong convective storm.

Testud et al. (2000) suggested the use of differential propagation phase to estimate the PIA in their so called ZPHI algorithm. In the last years this powerful method was slightly improved by several authors, e.g. Bringi et al. (2001), Gorgucci and Baldini (2007).

C. In-operational radar calibration

To achieve reliable accuracy for rain rate measurements based on reflectivity measurements the reflectivity Z_{hh} should be determined with an accuracy up to 1 dBZ and the differential reflectivity Z_{dr} up to 0.2 dB, cf. Illingworth (2003). To fit these needs an in-operational radar calibration is useful.

Calibration methods based on differential propagation phase measurements were presented in Goddard et al. (1994) and Gorgucci et al. (1999). Both calibration methods are similar

and work only in regions where attenuation is negligible so at S-band, and for weak rain events also at C-band.

D. Further applications

As suggested by Seliga and Bringi (1978) the differential propagation phase maybe also useful as additional parameter to estimate the parameters of the drop-size distribution. This is especially necessary when a three-parameter drop-size distribution, Ulbrich (1983), is used instead of the usual two-parameter exponential distribution, Marshall and Palmer (1948).

Because ground clutter affects the measurement of the differential propagation phase by adding a random differential backscatter phase the check of the standard deviation of ϕ_{dp} may support the detection of ground clutters and anomalous propagation, Zrnić and Ryzhkov (1996).

The differential propagation phase may also be used for hydrometeor classification algorithms as shown by Straka et al. (2000) and to estimate the rainwater content of a storm, Jameson (1994).

VI. CONCLUSION

In the last 30 years lots of research was done to make use of the polarisation of electromagnetic waves in the field of remote sensing. After it is been successfully used in synthetic aperture radars, polarimetry becomes finally operational for weather radars whereas in communications still a lot of research is necessary to enhance the channel capacity using polarimetry.

Especially the differential propagation phase will have an huge impact on improving the data quality for more accurate rain rate measurements. A direct use of the differential propagation phase to estimate the rain rate in operational weather radars is not probable due to its lack of resolution and the noisiness of the K_{dp}^{1-way} estimates.

After the boom of algorithms using K_{dp}^{1-way} in the 1990s the current trend seems the development of algorithms based

directly on the differential propagation phase ϕ_{dp} . This is the direction for the ongoing research on the differential propagation phase in polarimetric weather radars.

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