Phenomenological theories of magnetic multilayers and related systems

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Abstract

In this thesis multidomain states in magnetically ordered systems with competing long-range and short range interactions are under consideration. In particular, in antiferromagnetically coupled multilayers with perpendicular anisotropy unusual multidomain textures can be stabilized due to a close competition between long-range demagnetization fields and short-range interlayer exchange coupling. These spatially inhomogeneous magnetic textures of regular multidomain configurations and irregular networks of topological defects as well as complex magnetization reversal processes are described in the frame of the phenomenological theory of magnetic domains. Using a modified model of stripe domains it is theoretically shown that the competition between dipolar coupling and antiferromagnetic interlayer exchange coupling causes an instability of ferromagnetically ordered multidomain states and results in three possible ground states: ferromagnetic multidomain state, antiferromagnetic homogeneous and antiferromagnetic multidomain states. The presented theory allows qualitatively to define the area of existence for each of these states depending on geometrical and material parameters of multilayers. In antiferromagnetically coupled superlattices with perpendicular anisotropy an applied magnetic bias field stabilizes specific multidomain states, so-called metamagnetic domains. A phenomenological theory developed in this thesis allows to derive the equilibrium sizes of metamagnetic stripe and bubble domains as functions of the antiferromagnetic exchange, the magnetic bias field, and the geometrical parameters of the multilayer. The magnetic phase diagram includes three different types of metamagnetic domain states, namely multidomains in the surface layer and in internal layers, and also mixed multidomain states may arise. Qualitative and quantitative analysis of step-like magnetization reversal shows a good agreement between the theory and experiment.

Analytical equations have been derived for the stray field components of these multidomain states in perpendicular multilayer systems. In particular, closed expressions for stray fields in the case of ferromagnetic and antiferromagnetic stripes are presented. The theoretical approach provides a basis for the analysis of magnetic force microscopy (MFM) images from this class of nanomagnetic systems. Peculiarities of the MFM contrast have been calculated for realistic tip models. These characteristic features in the MFM signals can be employed for the investigations of the different multidomain modes.

The methods developed for stripe-like magnetic domains are employed to calculate magnetization processes in twinned microstructures of ferromagnetic shape-memory materials. The remarkable phenomenon of giant magnetic field induced strain transformations in such ferromagnetic shape memory alloys as Ni-Mn-Ga, Ni-Mn-Al, or Fe-Pd arises as an interplay of two physical effects: (i) A martensitic transition creating competing phases, i.e. crystallographic domains or variants, which are crystallographically equivalent but have different orientation. (ii) High uniaxial magnetocrystalline anisotropy that pins the magnetization vectors along certain directions of these martensite variants. Then, an applied magnetic field
can drive a microstructural transformation by which the martensitic twins, i.e. the different crystallographic domains, are redistributed in the martensitic state.

Within the phenomenological (micromagnetic) theory the equilibrium parameters of multi-variant stripe patterns have been derived as functions of the applied field for an extended single-crystalline plate. The calculated magnetic phase diagram allows to give a detailed description of the magnetic field-driven martensitic twin rearrangement in single crystals of magnetic shape-memory alloys. The analysis reveals the crucial role of preformed twins and of the dipolar stray-field energy for the magnetic-field driven transformation process in magnetic shape-memory materials.

This work has been done in close collaboration with a group of experimentalists from Institute of Metallic Materials of IFW Dresden, Germany and San Jose Research Center of Hitachi Global Storage Technologies, United States. Comparisons between theoretical and experimental data from this cooperation are presented throughout this thesis as vital part of my work on these different subjects.
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Introduction

Multidomain states considerably influence physical properties of condensed matter systems with spontaneous polarization. Such spatially inhomogeneous patterns form ground states of ferromagnetic [1, 2], ferroelectric [3], or ferroelastic [4] films. Recently multidomain structures have been observed in nanoscale magnetic films and multilayers with strong perpendicular anisotropy [5–9] and in ferroelectric superlattices [3, 10, 11]. Similar spatially modulated states can also arise in polar or magnetic liquid crystals [12, 13], polar multiblock copolymer layers, [14] in superconducting films or magnetic-superconductor hybrids, [15] and in shape memory alloy films [16, 17].

Multilayer systems with perpendicular polarization components provide ideal experimental models to investigate fundamental aspects of ordered structures and stable pattern formation in confining geometries. Control of such regular depolarization patterns is also of practical interest. In particular, nanoscale superlattices of antiferromagnetically coupled ferromagnetic layers have already become components of magnetoresistive devices. Antiferromagnetically coupled [Co/Pt(or Pd)]/Ru(or Ir), Co/Ir, Fe/Au, [Co/Pt]/NiO superlattices with strong perpendicular anisotropy [6–8] are considered as promising candidates for nonvolatile magnetic recording media, spin electronics devices, high-density storage technologies, and other applications [18]. According to recent experiments [3, 6, 7] due to the strong competition between antiferromagnetic interlayer exchange and magnetostatic couplings these nanoscale superlattices are characterized by novel multidomain states, unusual depolarization processes, and other specific effects [6, 7, 19, 20] which have no counterpart in other layered systems with perpendicular polarization [2].

In this thesis a phenomenological theory of multidomain states in antiferromagnetically coupled multilayers with strong perpendicular anisotropy and shape memory alloy plates is presented. The peculiarities of magnetization processes, effects of configuration hysteresis, wide variability of magnetic field-driven reorientation transitions and corresponding multidomain states are the subject of study in this work. The theoretical formulations of these problems employ the known phenomenological framework of micromagnetism and magnetic domain theory. Corresponding theories for other ferroic systems could be worked out in a similar fashion. For historical and practical reasons, domain theory of the kind employed in this thesis is probably best developed in the field of magnetism. Therefore, the parallel and equivalent formulations for other ferroic systems are not discussed in this text.

A major aim of this thesis is a complete overview of phase diagrams and domain state evolution under applied fields in magnetic superlattices with perpendicular magnetization and antiferromagnetic interlayer exchange couplings. This choice of system was motivated by the current strong interest in such artificial antiferromagnets and the great number of experimental studies. As central tool of these investigations and practical calculations, a method for the analytical evaluation of the dipolar stray field energies has been achieved for stripe-like and other domain states in magnetic multilayer systems or superlattices with arbitrary
geometrical parameters. This type of stripe-domain model for single layers and multilayers has venerable foundations dating back to Kittel [1]. The method developed here has the advantage that it yields closed expressions for the slowly converging dipolar energy contributions owing to the long-range of the magnetostatic stray fields. Therefore, the method could be used to calculate in full detail magnetic phase diagrams for domain states in multilayers without recourse to approximate or numerical schemes, while approximate evaluations can be introduced and investigated in a controlled manner.

The thesis is organized as follows. Each chapter begins with a brief introduction and has a short description of its organization.

In Chapter 1, the theoretical background of micromagnetism and domain theory is presented with the detailed discussion of essential energy terms. We give particular consideration to interlayer exchange interaction and surface induced anisotropy. In this chapter we also describe the theory of stripe and bubble domains in thin ferromagnetic single layer with high perpendicular anisotropy introduced by C. Kittel [1] and developed by Z. Malek, V. Kambersky [21] and independently by C. Kooy, U. Enz [22]. Here, the original method for the rigorous solution of the magnetostatic problem is introduced based on integral representation of the stray-field energy.

In Chapter 2 we give the detailed description of this approach for multidomain states in multilayers with perpendicular anisotropy and interlayer exchange coupling. In particular, we extend the integral representation of the magnetostatic energy introduced in Chapter 1 to the case of multilayer and introduce a model of \textit{shifted} ferro stripes in exchange coupled multilayers. This method now allows us to derive a number of new and rigorous results for domain states in magnetic multilayer systems. Within the scope of the model of \textit{shifted} ferro stripes we describe instabilities of stripe domain patterns in antiferromagnetically coupled multilayers. We also discuss peculiarities of phase diagrams for ground states in multilayers with even and odd number of magnetic layers, field induced transitions and in-field evolution of so-called metamagnetic domains. We qualitatively and quantitatively compare our results with experimental data on magnetization reversal and magnetic force microscopy images.

In Chapter 3 we discuss application of our theory in magnetic force microscopy for the experimental study on domain structure in antiferromagnetically and ferromagnetically coupled multilayer. In this part, a number of useful approximations for the rapid calculation of the stray fields above a sample displaying different stripe-like domains are discussed. They will become useful for detailed quantitative evaluations of magnetic force microscopy data.

In Chapter 4 we devote to the application of the stripe domain theory to twinned microstructures in ferromagnetic shape memory alloy. This part is an adaption of the theory of stripe-like domains for crystallographically twinned ferromagnetic systems with a particular orientation of walls between magnetic domains, that are equivalent to fully magnetized twins. Calculations for our model of a ferromagnetic shape memory single crystal plate show that the dipolar stray field energy plays a crucial role for the redistribution process of crystallographic twin variants under an applied field, which is the celebrated magnetic shape-memory effect.

Conclusion and outlook on the problems presented in this thesis are given in the end of the thesis.
1 Phenomenological Theory of Multidomain States in Thin Ferromagnetic Layers

In the first section of this chapter a short introduction to micromagnetism and domain theory is given. The second section is devoted to the main contributions to the free energy of magnets. We give more detailed theoretical description of the interlayer exchange coupling separately from other contributions because it plays a very important role in formation of the domain structures in magnetic multilayers which is the subject of study of this thesis. Then we briefly discuss the origin of domains in thin ferromagnetic layers and calculation of a structure and energy for the simplest type of domain walls. Finally in sixth section we apply the domain theory approach for a detailed analysis of stripe and bubble domains in thin ferromagnetic single layers.

1.1 Domain Theory

The basic concepts of the theory of magnetism are terms like spin, magnetic moment, and magnetization. They are usually connected to different length scales on which the magnetic properties are considered. Magnetism on these different length scales is generally described by different theoretical frameworks. The smallest, most detailed level to study the magnetic properties of solids is their electronic structure. In most general cases exactly the spin magnetic moment as well as the angular magnetic moment of the electrons are ultimately responsible for large-scale phenomena connected with macroscopic domain structures. The largest microscopic coupling term is exchange coupling of electrons with equal spins. The electron spin can thus be regarded as the fundamental entity of magnetism in solids. This “electronic” level of description is governed by the quantum theory of solids. It is obviously not possible to describe an entire ferromagnetic particle including its magnetic domain structure on a purely electronic level, even if the element is only a few 100 nm large. In fact, theoretical studies on an electronic level often require several simplifications, such as the approximation of periodic boundary conditions, which states that it is sufficient to consider one elementary cell and construct a magnetic solid by a repetition of such cells. Moreover the dipolar energy is usually neglected, because its energy density inside the solid is much weaker than the other energy terms involved in these calculations. Important achievements have been obtained with numerical simulations within the framework of the quantum theory of solids. Density functional theory made it possible to obtain material properties based on first-principles calculations [23]. However, it is obviously not possible to cover the wide range of relevant aspects of magnetism in solids with only one single theory. For example, the mentioned approximations prevent the consideration of such important aspects of ferromagnets...
as their stray-field energy, their magnetic domain structure or magnetostatic effects.

The next level of approximation is given by the atomistic theories, e.g., the Heisenberg model. In this approximation each atom of the magnet is assumed to carry a magnetic moment, and those magnetic moments are interacting with each other in the lattice of the solid. The magnetic moment $\mu_i$ originating in the electron spin is ascribed to the atom position $R_i$ in this model, caused by a local spin of fixed magnitude $S^2$ and only its quantized direction is retained as degree of freedom. The exchange interaction between $i$-th and $j$-th moments, $J_{ij}$, has a quantum-mechanical origin (Pauli principle), but is here assumed to be just a constant factor to the scalar product of these moments $(\mu_i \cdot \mu_j)$:

$$\mathcal{H} = \sum_{i \neq j} J_{ij}(\mu_i \cdot \mu_j)$$  \hspace{1cm} (1.1)

Extensions of the Heisenberg model can contain further energy terms like the anisotropy and the dipolar interaction. Various approximations are used in the atomistic Heisenberg models in order to address larger length scales. A common approximation used in calculations based on the Heisenberg model consists in considering only nearest-neighbour interactions, since the exchange interaction is short-ranged. The atomistic Heisenberg model can serve to describe spin structures on atomistic level, e.g., in monoatomic chains or ultrathin magnetic films and surfaces. In many cases, however, the relevant length scales for magnetic structures are much larger than the atomic lattice constants. For example, one of the most fundamental magnetization structures, the magnetic domain wall, typically extends over several tens of nanometres in bulk material.

The transition from the atomistic to the \textit{micromagnetic} approximation is characterized by a qualitative transition from a discrete to a continuous representation. In other words, the fundamental difference between the micromagnetic representation of ferromagnets and atomistic Heisenberg models lies in the fact that the magnetic structure is represented by a continuous vector field in the micromagnetic approximation, while it is considered as the ensemble of discrete magnetic moments in atomistic models. In this sense this qualitatively different approach involves a qualitative change of the equations describing the problem. In particular, in the micromagnetic approximation, the microscopic magnetic moments are replaced by an averaged quantity: the magnetization which is defined as the density of magnetic moments $M = N \mu / V$.

Here, $N$ is the number of magnetic moments in the sample of volume $V$. Correspondingly, the summations over dipoles or over magnetic moments occurring in atomistic representations are replaced by volume integrals containing the magnetic moment density, i.e., the magnetization. The vector field of the magnetization $\mathbf{M}$ is a ordered field, meaning that at any point in space $r$ it is free to assume any direction which it can also change in time. Solution of micromagnetic problems consists in calculating the vector field of the magnetization $\mathbf{M}(r)$ and sometimes also its temporal evolution $\partial \mathbf{M} / \partial t$.

The main principles of micromagnetic theory originate in the article of Landau and Lifshitz [24] published in 1935. Their theory is based on a variational principle: it searches for magnetization distributions with the smallest total energy. This variational principle leads to a set of integro-differential equations, the micromagnetic equations. They were given in [24] for one dimension. Then, W.F. Brown extended the equations to three dimensions [25–27], including fully the stray field effects [28]. Generally, a form of the Landau-Lifshitz-Gilbert
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The equation is

$$\frac{\partial \mathbf{M}}{\partial t} = -|\gamma| \mathbf{M} \times \mathbf{H}_{\text{eff}} + \alpha \frac{\mathbf{M}}{M_S} \times \frac{\partial \mathbf{M}}{\partial t},$$

(1.2)

where $\gamma$ is the electron gyromagnetic ratio and $\alpha$ is the Gilbert phenomenological damping parameter. The effective field $\mathbf{H}_{\text{eff}}$ is defined as the negative variational derivative of the total micromagnetic energy density $e_{\text{tot}}$ with respect to the magnetization:

$$\mathbf{H}_{\text{eff}} = -\frac{\delta e_{\text{tot}}}{\delta \mathbf{M}}.$$  

(1.3)

This definition of the effective field can be compared with a similar definition used in mechanical systems, where the local force density can be obtained as negative gradient of the energy density. The effective field contains all effects from external and internal fields or energy contributions which will be discussed in Sect. 1.2. A formal derivation of the effective field can be found in textbooks of W.F. Brown Jr. (see e.g. Refs. [26] and [27]).

Equation Eq. (1.2) can be shown to be equivalent to the more complicated form

$$\frac{\partial \mathbf{M}}{\partial t} = -\frac{|\gamma|}{1 + \alpha^2} \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{|\gamma| \alpha}{1 + \alpha^2} \frac{\mathbf{M}}{M_S} \times \mathbf{M} \times \mathbf{H}_{\text{eff}}.$$  

(1.4)

Originally, in 1935, Landau and Lifshitz used expression (1.4) without the denominator $(1 + \alpha^2)$, which arose from Gilbert’s modification in 1955 [29].

The micromagnetic equations are complicated non-linear and non-local equations; they are therefore difficult to solve analytically, except in cases in which a linearization is possible. Typical magnetic structures studied in the framework of micromagnetism are magnetic domain walls, magnetic vortices and domain patterns in mesoscopic ferromagnets, but also dynamic effects like spin waves, magnetic normal modes and magnetization reversal processes [2, 27, 30]. To treat such problems, work on numerical solutions of the micromagnetic equations is increasingly pursued. However, it is quite difficult to use numerical methods of finite elements or finite differences for microscopic bodies with characteristic dimension $l \gg 1 \mu \text{m}$. The domain theory is a theory that combines discrete, uniformly magnetized domains with the results of micromagnetism for the connecting elements, the domain walls and their substructures. The domain theory can be regarded as the next largest scale for the theoretical description of ferromagnets.

The main principles of domain theory as well as micromagnetic theory are based on the same article of Landau and Lifshitz [24]. Their presently accepted form was introduced by Kittel [1]. The approximation of the domain theory is to a certain extent similar to the theory of micromagnetics. The results deduced using approximations of domain theory have, by-turn, played a very important role for the development of micromagnetics.

The domain theory considers that the magnetic structure of a ferromagnet is subdivided into magnetic domains (regions within a magnetic material which has uniform magnetization), which are separated by domain walls. Domain walls describe the magnetic structure in the transition region which is localized in a confined space between two magnetic domains. However, on the length scale relevant for domain theory, the details of the transition regions in which the magnetization changes its direction (domain walls or vortices) are neglected. The
magnetization is assumed to change its direction abruptly between neighbouring domains. The transition regions are treated as infinitely thin entities (lines or interfaces). Their contribution to the total energy is considered by assigning an exchange energy density to them, such that the total exchange energy of the sample results as integration over the domain wall area. However to define the domain wall energy density and corresponding rotation of the magnetization between two domains one should solve a complex variational problem. In Sect. 1.6 we discuss the simplest case of such variational problems.

As it was established initially by Landau and Lifshitz [24] domain theory as well as micromagnetic theory are based on the same variational principle which is derived from thermodynamic principles. According to this principle, in a static equilibrium state, the magnetization field \( M(\mathbf{r}) \) arranges such as to minimize the total energy but it preserves everywhere its magnitude, \( |M| \equiv M_s \). The saturation magnetization \( M_s \) (sometimes also called spontaneous magnetization) is a material property which is characteristic for ferromagnetic materials. The rare cases in which the saturation magnetization \( M_s \) varies inside a micromagnetic configuration are ignored in this theory. This energy minimum can either be a local or a global minimum. If several different minima can be achieved, the magnetic history of the sample is decisive for the selection of the equilibrium configurations.

The procedure used to describe domain structures of a studied system within the scope of the domain theory is described in detail in Ref. [2] (see p. 99). Firstly, we find reasonable domain models that are compatible with the observed pattern. Then we calculate their energies and choose the best for further analysis. Vary the parameters of the chosen model (angles, lengths etc.) continuously, again looking for the lowest energy. If the calculated configuration is still consistent with observation, the model may be assumed to be correct; if not, other models must be checked for lower energy and better agreement with the experiment. This seemingly involved procedure is necessary because as it was already mentioned above a direct numerical solution of the micromagnetic equations is impossible for all but extreme cases such as micron-scale thin film elements. In all other cases only domain theory can support domain observation, and the continuum theory of micromagnetics can only help in supplying the necessary elements.

1.2 Overview of Magnetic Energy Terms

The magnetization-dependent contributions to the energy are the starting point of domain theory. In micromagnetic problems, the most important contributions to the total energy are usually the ferromagnetic exchange energy, the dipolar interaction energy (or so-called magnetostatic energy), the magneto-crystalline anisotropy energy and the Zeeman energy in an external magnetic field. These energy terms will be briefly described in the next subsections. A more detailed discussion of the micromagnetic energy terms can be found in various textbooks on micromagnetism [2, 27, 30, 31]. Strictly speaking the total energy of a ferromagnet has in addition contributions of magnetoelastic effect namely magnetostriction. However, because magnetoeelastic effects can be ignored in most practical cases we do not discuss them here.

First, we must distinguish between local and non-local magnetic energy terms. The local terms (e.g. the anisotropy energy and the applied field (Zeeman) energy) are based on energy densities, which are given by the local values of the magnetization direction only.
Their integral value is calculated by a simple integral of the form $E_{\text{loc}} = \int f(m) dV$ over the sample, where the energy density function $f(m)$ is a function of the magnetization direction $m$. The exchange energy can be treated as local in a sense that it is calculated by an integral over a function of the derivatives of the magnetization direction. In particular for an isotropic exchange it leads to the integration over squared gradients of the magnetization direction.

The non-local energy contributions stem from the stray field energy. This energy term gives rise to torques on the magnetization vector that depend at any point on the magnetization directions at every other point. Non-local energy terms cannot be calculated by a single integration. For example the stray field energy may be calculated by the following procedure [2]: firstly, a scalar magnetic potential is derived from integration over so-called magnetic “charges”, the sinks and sources of the magnetization vector field. A second integration over the product of charges and potential then leads to the total energy. In this approach the stray field is integrated for every given magnetization distribution, looking for the minimum energy as a function of the magnetization field only. It is precisely the non-local terms that make micromagnetics and domain analysis both interesting, and complicated.

### 1.2.1 Exchange Energy

The fundamental property of a ferromagnet (or a ferrimagnet) is their tendency to keep neighbouring magnetic moments parallel to each other. The short-range exchange interaction prevents strong inhomogeneities of the magnetization on small length scales. In other words, any increase of inhomogeneity of the magnetization field increases the exchange energy which can be described by the “stiffness” expression [24]:

$$E_{\text{ex}} = A \int \left( \nabla m_x^2 + \nabla m_y^2 + \nabla m_z^2 \right) dV \quad (1.5)$$

where $A$ is so-called exchange stiffness constant and $m = M/M_s$ is the reduced or normalized magnetization. It should be noted that $A$ and $M_s$ are in general temperature dependent. This expression can also be derived from a Taylor expansion of the isotropic Heisenberg interaction Eq. (1.1) assuming small-angle deviations between neighbouring spin magnetic moments $\mu_i$ and $\mu_j$ [30,31]. The exchange energy Eq. (1.5) is called isotropic because it is independent of the direction of the change relative to the magnetization direction. The order of magnitude of the exchange constant $A$ is about $10^{-7} - 2 \times 10^{-6}$ erg/cm [$10^{-12} - 2 \times 10^{-11}$ J/m]. The value $A$ for cobalt in thin layer is usually about $2 \times 10^{-6}$ erg/cm [2].

### 1.2.2 Anisotropy Energy

Magnetic anisotropy originates from the dependence of magnetic energy on the relative direction between the magnetization vector and the atom configuration $R_i$. Basically, the two main sources of the magnetic anisotropy are the magnetic dipolar interaction and the spin-orbit interaction. Due to its long range character, the dipolar interaction generally results in a contribution to the anisotropy via stray-fields, which depends on the shape of the specimen (shape anisotropy). It is of particular importance in thin films, and is largely responsible for the in-plane magnetization usually observed. In the absence of spin-orbit and dipolar interaction, the total energy of the electron-spin system does not depend on the direction of the magnetization. In a simplified picture the spins are coupled via the spin-orbit interaction.
to the orbits which, in turn, are influenced by the crystal lattice. For conduction electrons
the spin-orbit interaction induces a coupling between spin momentum and orbital momentum,
which then couples the total (spin plus orbital) magnetic moment to the crystal axes. This
coupling results in a total energy which depends on the orientation of the magnetization
relative to the crystalline axes. It is precisely this coupling that reflects the symmetry of
the crystal. According to the crystal symmetry, the direction of the magnetization favours energetically an alignment towards certain axes. Hexagonal and tetragonal crystals show a uniaxial anisotropy which is phenomenologically described up to fourth-order terms

\[ e_u = K_{u1} \left( 1 - (\mathbf{m} \cdot \mathbf{a})^2 \right) + K_{u2} \left( 1 - (\mathbf{m} \cdot \mathbf{a})^2 \right)^2 \]  

where \( K_{u1} \) and \( K_{u2} \) are the uniaxial anisotropy constants and \( \mathbf{a} \) is a unit vector parallel
to the anisotropy axis. The case of a large positive \( K_{u1} \) corresponds to an easy axis while large negative \( K_{u1} \) corresponds to easy plane perpendicular to the anisotropy axis. The easy axis or
easy plane represents the preferential orientation of the magnetization in the crystal. For
intermediate value \((0 > K_{u1}/K_{u2} > -2)\) the easy directions lie on a cone with the angle
\( \theta \) relative to the axis given by \( \sin^2 \theta = -\frac{1}{2} K_{u1}/K_{u2} \). The three different cases are called uniaxial, planar and conical magnetic anisotropy. 
Herein we will deal only with uniaxial anisotropy case when \( K_{u1} >> K_{u2} \) and the contribution from fourth-order terms can be neglected. More details about cubic, orthorhombic or exchange anisotropy can be found in Hubert and Schäfer’s textbook on magnetic domains [2].

Here we should give careful consideration to another anisotropic energy term that applies only to the surface magnetization which play extremely important role in magnetic thin layers and multilayers. As was first pointed out by Néel [32] the lowered symmetry at an interface strongly modifies the contribution of magnetocrystalline anisotropy as compared to the bulk, yielding, a so-called interface anisotropy. In conjunction with the overlap in wave functions between neighbouring atoms, the spin-orbit interaction is also responsible for the magnetooelastic or magnetostrictive anisotropy induced in a strained system, a situation which is frequently encountered in multilayers due to the lattice mismatch between the adjacent layers.
The more detailed discussion of each of these anisotropy terms is given in Ref. [33] where also a review of experiment is given.

First we introduce the simplest phenomenological approach for the surface anisotropy. This approach is based on concept of the effective magnetic anisotropy energy which could be separated in a volume contribution \( K_v \) (erg/cm\(^3\) or J/m\(^3\)) and a contribution from the interfaces \( K_s \) (erg/cm\(^2\) or J/m\(^2\)) and approximately obeys the relation

\[ K_u = K_{\text{eff}} = K_v + 2K_s/t. \]  

This relation just represents a weighted average of the magnetic anisotropy energy of the
interface atoms and the inner atoms of a magnetic layer of thickness \( t \). Equation (1.7) is commonly used in experimental studies, and the determination of \( K_v \) and \( K_s \) can be obtained by a plot of the product \( K_{\text{eff}} \) versus \( t \). Figure 1.1 shows a typical example of such a plot for Co/Pd multilayers [34]. A positive \( K_{\text{eff}} \) describes the case of a preferred direction of the magnetization perpendicular to the layer plane. The negative slope indicates predominance of negative volume anisotropy \( K_v \), favouring in-plane magnetization, while the intercept at zero Co thickness indicates positive interface anisotropy \( K_s \), favouring perpendicular magnetization. Below a certain thickness \( t_\perp = -2K_s/K_v \) (in this case 13 Å) the interface anisotropy contribution

\[ e_u = K_{u1} \left( 1 - (\mathbf{m} \cdot \mathbf{a})^2 \right) + K_{u2} \left( 1 - (\mathbf{m} \cdot \mathbf{a})^2 \right)^2 \]  

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Here we should give careful consideration to another anisotropic energy term that applies only to the surface magnetization which play extremely important role in magnetic thin layers and multilayers. As was first pointed out by Néel [32] the lowered symmetry at an interface strongly modifies the contribution of magnetocrystalline anisotropy as compared to the bulk, yielding, a so-called interface anisotropy. In conjunction with the overlap in wave functions between neighbouring atoms, the spin-orbit interaction is also responsible for the magnetooelastic or magnetostrictive anisotropy induced in a strained system, a situation which is frequently encountered in multilayers due to the lattice mismatch between the adjacent layers.
The more detailed discussion of each of these anisotropy terms is given in Ref. [33] where also a review of experiment is given.

First we introduce the simplest phenomenological approach for the surface anisotropy. This approach is based on concept of the effective magnetic anisotropy energy which could be separated in a volume contribution \( K_v \) (erg/cm\(^3\) or J/m\(^3\)) and a contribution from the interfaces \( K_s \) (erg/cm\(^2\) or J/m\(^2\)) and approximately obeys the relation

\[ K_u = K_{\text{eff}} = K_v + 2K_s/t. \]  

This relation just represents a weighted average of the magnetic anisotropy energy of the
interface atoms and the inner atoms of a magnetic layer of thickness \( t \). Equation (1.7) is commonly used in experimental studies, and the determination of \( K_v \) and \( K_s \) can be obtained by a plot of the product \( K_{\text{eff}} \) versus \( t \). Figure 1.1 shows a typical example of such a plot for Co/Pd multilayers [34]. A positive \( K_{\text{eff}} \) describes the case of a preferred direction of the magnetization perpendicular to the layer plane. The negative slope indicates predominance of negative volume anisotropy \( K_v \), favouring in-plane magnetization, while the intercept at zero Co thickness indicates positive interface anisotropy \( K_s \), favouring perpendicular magnetization. Below a certain thickness \( t_\perp = -2K_s/K_v \) (in this case 13 Å) the interface anisotropy contribution

outweighs the volume contribution, resulting in a perpendicularly magnetized system. Such behaviour is a general feature of all magnetic multilayers composed on Co/Pt(Pd), Co/Cu, Fe/Au or Ni/Cu bilayers.

It should be noted that this simplest approach cannot clarify some feature of these films. In particular, one of them is an “anomalous” behaviour of the magnetic anisotropy. The effective magnetic anisotropy of films with induced out-of-plane anisotropy depends nonlinearly and nonmonotonically on film thickness [34]. This is at variance with the predictions of Néel’s theory [32] and in contrast to the behaviour of many multilayer systems. In Ref. [35] a new phenomenological theory that gives a consistent description of the induced magnetic anisotropy in perpendicular magnetic layers has been introduced. In this approach it is assumed that \( K(r) \) is homogeneous in the magnetic layer plane and varies only along the normal to the surface assuming certain fixed values on free surfaces and interfaces \( K(0) = K(1) \) on upper surface and \( K(0) = K(2) \) on lower surface of magnetic layer. For these boundary conditions the minimization of a functional which has the standard form of a Landau- Ginzburg theory for phase transitions (for detail see Ref. [35, 36]) yields the analytical solution for \( K(z) \). Experimentally obtained anisotropy data usually are presented as a product of the total anisotropy \( K_{\text{eff}} \) and the layer thickness \( t \). Thickness dependencies of the product \( K_{\text{eff}}t \) is linear for most layered systems [37, 38]. However, this is not the case for perpendicular magnetized films (see Fig. 1.1, b and Refs. [34, 39, 40]). Within such an approach \( K_{\text{eff}}t \) is calculated by integration of \( K(z) \) over the layer thickness

\[
K_{\text{eff}}t = \int_0^t (K(z) - 2\pi M_s^2) \, dt. \tag{1.8}
\]

The value of perpendicular anisotropy which can be achieved for example in Co/Pt(Pd) multilayer is about several Merg/cm³ (0.1 MJ/m³) and can be precisely tuned by varying...
the thicknesses of magnetic and nonmagnetic layer [33]. For example for a (111)-oriented [Co(4.5 A)/Pt(10 A)]ₙ multilayer with \( n = 10 \) a perpendicular magnetized ground state was found with \( K_v = 7.0 \times 10^6 \text{ erg/cm}^3 \) and \( K_s = 0.27 \text{ erg/cm}^2 \) [41]. The corresponding value of effective uniaxial anisotropy (see Eq. 1.7) \( K_{\text{eff}} = 6.5 \times 10^6 \text{ erg/cm}^3 \) is in good agreement with the \( K_{\text{eff}} = 5 \times 10^6 \text{ erg/cm}^3 \) for a (111)-oriented [Co(4 A)/Pt(7 A)]ₙ multilayer system which we use as a model system for comparison between theoretical and experimental results in following chapters.

### 1.2.3 External Field (Zeeman) Energy

The magnetic field energy can be separated into two parts, the external field energy and the stray field energy. The first part, the interaction energy of the magnetization vector field with an applied external field \( H \) (Zeeman energy) is simply:

\[
E_H = -M_s \int \mathbf{H} \cdot \mathbf{m} \, dV
\]  

(1.9)

For a uniform external field this energy depends only on the average magnetization and not on the particular domain structure or the sample shape.

### 1.2.4 Stray Field Energy

The second part of the magnetic field energy is connected with the magnetic field generated by the magnetic body itself. It arises because each magnetic moment in a ferromagnetic sample represents a magnetic dipole and therefore contributes to a total magnetic field \( H_d \) inside the sample. The energy connected with this field is known as the stray field energy or the magnetostatic energy.

Starting from Maxwell’s equation \( \text{div}\, \mathbf{B} = \text{div}(\mathbf{H} + 4\pi \mathbf{M}) = 0 \), we define as the stray field \( H_d \) the field generated by the divergence of the magnetization \( \mathbf{M} \):

\[
\text{div}\, H_d = -\text{div}(4\pi \mathbf{M}).
\]  

(1.10)

The local stray field energy density depends on the orientation of the magnetic moments with respect to this field

\[
e_d = -\frac{1}{2}(H_d \cdot \mathbf{M}).
\]  

(1.11)

In the literature, different terms are used for the field \( H_d \). It is either called the magnetic stray field, the dipolar field, the demagnetizing field or the magnetostatic field. The factor 1/2 in Eq. (1.11) is required for self-energy terms. This energy contribution arises from the long-range magnetostatic interaction between the magnetic moments in the sample. The sinks and sources of the magnetization act like positive and negative “magnetic charges” for the stray field. The field can be calculated like a field in electrostatics from the electrical charges. The only difference is that magnetic charges never appear isolated but are always balanced by opposite charges. The total magnetostatic energy of the system is:

\[
E_d = \frac{1}{2} \int_{\text{all space}} H_d^2 \, dV = -\frac{1}{2} \int_{\text{all sample}} (H_d \cdot \mathbf{M}) \, dV.
\]  

(1.12)
Figure 1.2: (Color) This sketch illustrates the origin of magnetic charges at an interface (a) and surfaces (b) of magnetically ordered media. The magnetic charge density at an interface equals \( \sigma = m_1 \cos \theta_1 - m_2 \cos \theta_2 \) and at an surface \( \sigma_1 = m_1 \cos \theta_1 \) and \( \sigma_2 = -m_2 \cos \theta_2 \).

The first integral extends over all space; it shows that the stray field energy is always positive, and is only zero if the stray field itself is zero everywhere. The second integral is mathematically equivalent for a finite sample. It is often easier to evaluate, since it extends only over the magnetic sample. A general solution of the stray field problem is given by potential theory. The reduced volume charge density \( \lambda \) and the surface charge density \( \sigma \) are defined in terms of the reduced magnetization \( \mathbf{m}(r) = \mathbf{M}(r)/M_s \):

\[
\lambda = - \text{div}(\mathbf{m}), \quad \sigma = (\mathbf{m} \cdot \mathbf{n}) \tag{1.13}
\]

where \( \mathbf{n} \) is the outward directed surface normal. If the body contains interfaces separating two media 1 and 2 (Fig. 1.2, a) with different magnetization \( \mathbf{m}_1 \) and \( \mathbf{m}_2 \) at the interface, then the interface charges \( \sigma = (\mathbf{m}_1 - \mathbf{m}_2) \cdot \mathbf{n} \) are formed (assuming that the interface normal \( \mathbf{n} \) points from medium 1 to medium 2). The surface charges of (1.13) are a special case of interface charges where the second medium is non-magnetic (Fig. 1.2, b).

With these quantities the potential of the stray field at position \( r \) is given by an integration over \( r' \):

\[
\Phi(r) = M_s \left[ \int \frac{\lambda(r')}{|r - r'|} dV' + \int \frac{\sigma(r')}{|r - r'|} dS' \right]. \tag{1.14}
\]

Another way of obtaining the expression for the magnetostatic potential \( \Phi(r) \) is the general solution of the Poisson equation

\[
\Delta \Phi_d(r) = -\lambda(r) = \text{div}(4\pi M). \tag{1.15}
\]

Then the stray field can be derived by \( H_d(r) = -\text{grad} \Phi(r) \). Another integration immediately yields the stray field energy:

\[
E_d = \frac{M_s}{2} \left[ \int \lambda(r)\Phi(r) dV + \int \sigma(r)\Phi(r) dS \right] \tag{1.16}
\]

The stray field energy calculation therefore amounts to a six-fold integration if volume charges \( \lambda \) are present otherwise if only surface charges \( \sigma \) are present it reduces to a four-fold integration. Although the integrand diverges at \( r = r' \), the integrals remain finite.
1 Phenomenological Theory of Multidomain States in Thin Ferromagnetic Layers

Figure 1.3: (Color) Sketch of an antiferromagnetically coupled bilayer separated by a non-
magnetic layer. The thicknesses of lower and upper magnetic layers are \( L_A \) and \( L_B \) and the thickness of nonmagnetic layer is \( t \). a) and b) correspond to the case of in-plane \( (K_u = (K_x, 0, 0)) \) and perpendicular \( (K_u = (0, 0, K_z)) \) uniaxial anisotropy at zero external magnetic field; c) large enough external magnetic field \( H \) align the magnetizations parallel.

1.3 Interlayer Exchange Coupling

Magnetic multilayers, in which magnetic layers are separated by nonmagnetic spacer layers, exhibit effect of interlayer exchange coupling (IEC) which was first observed in 1986 for Dy and Gd films separated by Y interlayers and for Fe films separated by Cr interlayers [42]. For ferromagnetic films like those of Gd and Fe, the IEC stabilizes either collinear (parallel or antiparallel) or noncollinear alignment of the magnetizations on opposite sides of the interlayer. The actual alignment is also affected by other interactions like anisotropy or applied external magnetic field \( H \). Large enough fields \( H \) overcome the coupling and align the magnetizations parallel (Fig. 1.3). In 1990, it was established that the oscillation of the magnetic coupling between ferromagnetic and antiferromagnetic alignment, as a function of interlayer thickness, is a general phenomenon of transition metal ferromagnets separated by nonmagnetic interlayers [43]. The discovery in 1988 of the Giant Magnetoresistance (GMR) effect in the Fe/Cr system led to enhanced interest in the magnetic coupling in multilayers of transition metal ferromagnets because of the many applications of GMR.

1.3.1 Phenomenological Description

A phenomenological description of the coupling proposed to explain the experimental observations gives the interlayer coupling energy, \( E_J \), per unit area as:

\[
E_J = J_1 (m_1 \cdot m_2) + J_2 (m_1 \cdot m_2)^2.
\] (1.17)

Here \( m_1 \) and \( m_2 \) are the magnetization vectors at the interface, \( J_1 \) is the bilinear and \( J_2 \) is the “biquadratic” coupling constant. This series describes generalizations of the Heisenberg form of coupling energy.

In the literature various definitions of the parameters are in use. For example the Eq. (1.17) is often alternatively defined with a minus sign in front of \( J_1 \) and with a plus sign in front of \( J_2 \) [44]. Here we use the form of the interlayer coupling energy introduced in [45].

If \( J_1 \) is negative, it favours parallel (FM) orientation of the magnetization in the two media. If it is positive, an antiparallel (AF) alignment is preferred. A positive value of \( J_2 \) may lead
to 90° relative orientation if $J_1$ is small. The origin of the two coupling coefficients is quite different. The bilinear term is closely related to the corresponding volume exchange stiffness effect and derivable from the same quantum mechanical foundations which will be discussed below. In contrast, the biquadratic term is attributed to various microscopic mechanisms. For example, as reviewed by Slonczewski the biquadratic term can be related to interface roughness. A detailed description of biquadratic coupling can be found in Ref. [46]. For purpose of micromagnetism and domain theory the question of the nature of the coupling effects can be ignored. However, in order to clarify some important effects as oscillation of the sign of $J_1$ on interlayer thickness and thermal dependence of $J_1$ we briefly discuss the microscopic theory of IEC.

A broad spectrum of theoretical approaches has been employed so far to explain the spectacular phenomenon of IEC, in particular first principles or tight-binding total energy calculations [47–50], the Ruderman-Kittel-Kasuya-Yosida (RKKY) theory [51–55], a free-electron model [56–59] and the Anderson $s–d$ Mixing models [60,61]. At the beginning of the nineties, it has been shown that all the above models can be unified into a more general approach in which the interlayer exchange coupling is interpreted as a quantum size effect and described in terms of quantum interference of electrons at the interfaces between the non-magnetic spacer and the ferromagnetic layers. In the next sections we summarized this theory of Bruno and Slonczewski given in Refs [45,62,63].

### 1.3.2 The Quantum Well Model

This subsection discusses a simple model that shows why the Fermi surface of the spacer layer plays an important role in the coupling between the magnetic layers. To explain in
Figure 1.5: (Color) a) shows the probability for an electron incident from the well to reflect from a single step $R_s(E)$ and to reflect from the quantum-well structure $T_w(E)$ as functions of the energy. b) shows the change in the density of states due to presence of the well [62]. The bound-state levels marked by vertical arrows, together with transmission resonances marked by the indicated peaks of the continuous part of $\Delta n(E)$, form one smooth system of sharply defined energy values $E_\nu$, $(\nu = 1, 2, 3 \ldots)$ [46]. At negative energies, the curve has been reduced by a factor of 10 to fit it in the figure.

Minimal terms the exchange theory, let us first consider a one-dimensional degenerate Fermi gas of “spinless” particles overfilling a rectangular quantum-well potential (Fig. 1.4) [62]. All following reasoning are based on facts that are well known from elementary quantum mechanics. Let $q$ be the variable particle wave number and $Q$ constant Fermi wave number inside the well similarly $k$ and $K$ outside the well. The energy $E = q^2 - Q^2 = k^2 - K^2$ of a Schrödinger particle wave generally propagating through all three subregions of this system is measured from the Fermi potential in reduced units. According to these conventions, the crystalline potential term is $E_p = -Q^2$ inside the well and $E_p = -K^2$ outside. The kinetic energy term is respectively $q^2$ or $k^2$.

In the range $-Q^2 < E < -K^2$, a state reflects with probability one from each of the steps. In general the multiple reflections cancel each other and a state cannot exist. However, there exist a finite number $N_b$ of localized states at which the interference is constructive and bound states with sharp levels $E = E_\nu$, $(\nu = 1, 2, 3 \ldots N_b)$ result. These bound states consist of waves bouncing back and forth in the well with tails exponentially decaying into the asymptotic regions Fig. 1.4. At positive energies there are scattering states at all energies. These scattering states consist of a plane wave incident on the quantum well from either side, a reflected wave with reduced amplitude on the same side of the well, waves scattering in both directions in the well, and a transmitted wave on the other side of the well (see Fig. 1.4). An incident wave with $E > -K^2$ propagating rightward within the well scatters from interface $B$ with reflection coefficient $R(E) = (q - k)/(q + k)$ Fig. 1.5 a. The reflected part of this wave scatters again at interface $A$, and so on. For an infinite number of particular discrete values of $E = E_\nu$ $(\nu > N_b)$ at which an integer number of wavelengths fit inside the well the state undergoes increased multiple scattering in the well, and transmits with unit probability through the spacer.

The probability $T_w(E)$ for a state incident from either side to transmit through the well is shown in Fig. 1.5 (a). The reflection probability $R_w(E)$ is just one minus this probability. Since there are no states in the asymptotic regions with energies $E < -K^2$, the reflection and transmission probability are not defined for these energies. At low energies near $E = -K^2$
the reflection probability is one, decreasing to zero as the energy increases. The set of states consisting of the bound states, plus the scattering states incident from each side make up a complete set of states for this potential. To compute the energy required to fill these states up to some Fermi level, it is necessary to compute the change in the density of states for this structure with respect to the constituent materials. Figure 1.5 (b) shows the change $\Delta n = n(t, \mathcal{E}) - n(0, \mathcal{E})$ in state density brought about by creation of the well in the limit that the size of the asymptotic region, goes to infinity ($L_A$ and $L_B \to \infty$ see Fig. 1.3). Upward pointing arrows locate the set of $\delta$-function contributions due to the bound levels. At higher $\mathcal{E}$, the peaks in the dependence of $\Delta n$ on $\mathcal{E}$ mark the transmission resonances (compare Fig. 1.5 (a and b)). The total energy at $T = 0$ K of this one-dimensional solid is the integral of the energy over the occupied states

$$E = \int_0^\infty \mathcal{E} n(\mathcal{E})d\mathcal{E}. \quad (1.18)$$

Figure 1.6 shows its change $\Delta E = E(t) - E(0)$ versus $t$. It represents the signed area bounded by $-Q^2 < \mathcal{E} < 0$ and lying between the horizontal axis and the curve $\Delta n(\mathcal{E})$ indicated by shading in Fig. 1.5 (b), plus the bound-state energy $\sum_{N_b} \mathcal{E}_n$. According to the above quantum-well relation, one of the resonant peaks of $\Delta n(\mathcal{E})$, shown in Fig. 1.5 b), passes downward through the Fermi level $\mathcal{E} = 0$ each time $t$ increases by approximately $\pi/Q$. Each such passage causes the integral in Eq. (1.18) to execute an oscillation exhibited in Fig. 1.6.

Approaching the limits $t \to \infty$ and of weak reflections $R \ll 1$, one finds [45,63]:

$$\Delta E = \left(\hbar^2 Q R^2 / 2\pi m_e t\right) \sin (2Qt). \quad (1.19)$$

Here $R = (Q - K)/(Q + K)$ is the reflection coefficient at the Fermi level, and physical units (with $m_e$ is the electron mass) are employed. Note that only particle states near the Fermi level contribute to $\Delta E$. The dashed curve in Fig. 1.6 represents this equation, which differs drastically from the exact relation (solid curve) only within the first period of oscillation. In general, the accuracy of the asymptotic form depends on the details of the band structures of the two materials, in particular on the strength of the reflection. It should be noted that the oscillation as a function of distance with a period set by the Fermi surface is a general property of metals.

Figure 1.6: Total energy versus symmetric-well thickness for a one-dimensional spinless Fermi gas at $T = 0$ K. The solid curve is exact; the dashed curve is Eq. (1.19) [62].

\[\begin{array}{c}
\text{Thickness} \\
\end{array}\]

\[\begin{array}{c}
0.01 \\
0 \\
-0.01 \\
\end{array}\]

\[\begin{array}{c}
0 \\
5 \\
10 \\
\end{array}\]
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![Diagram](image)

Figure 1.7: Quantum wells for exchange coupling. In (a) the left (right) quantum well shows the potential seen by spin-up (-down) electrons in a ferromagnetically aligned quantum-well structure. In (b) the left (right) quantum well shows the potential seen by spin-up (-down) electrons in an antiferromagnetically aligned quantum-well structure [62].

1.3.3 Interlayer Exchange Coupling Due to Spin-dependent Reflectivity

For magnetic sandwich structures the oscillatory exchange coupling arises for the same reasons as the oscillatory energy in a quantum well. However, it is much easier to detect because it is much easier to measure the magnetic state of a material than its total cohesive energy. It is known that the Stoner exchange potential inside each ferromagnet is spin dependent. Model potentials for both spins are shown in Fig. 1.7 for the cases of ferromagnetic and antiferromagnetic alignment of the magnetizations. For each spin system the energy oscillates for the different magnetic configurations; the difference in these oscillatory energies is the oscillatory exchange coupling. There are three different oscillatory energies, one each for spin-up and spin-down electrons in a ferromagnetically aligned sample, and one for either spin in an antiferromagnetically aligned sample. All of the oscillatory energies have the same period because the period is determined by the Fermi surface of the bulk spacer-layer material, but since the potential barriers are different, the reflection probabilities are different, and hence the amplitudes of the oscillatory energies are different. Taking the difference between the sum of the energies for the spin-up and spin-down electrons in a ferromagnetically aligned sandwich and twice the energy for either spin in an antiferromagnetically aligned sandwich gives the exchange coupling

\[ J(t) = \left( \frac{h^2 Q}{2 \pi m_e} \right) \left[ |R^\uparrow|^2 + |R^\downarrow|^2 - |R^\uparrow R^\downarrow|^2 \right] \frac{1}{t} \sin(2Qt), \]  

(1.20)

where \( R^\uparrow \) (\( R^\downarrow \)) is the reflection amplitude for a spin-up (-down) electron in the well material reflecting from an up magnetization barrier, and \( R^\uparrow = R^\downarrow \) and \( R^\uparrow = R^\downarrow \). Thus, the exchange coupling found in magnetic sandwich structures has the same origin as all other oscillation behavior in metals, the response of the electrons at the Fermi surface.

If the interfaces between the magnetic material and the nonmagnetic material are coherent, that is, the materials share a common lattice net, then the momentum parallel to the interface is conserved when an electron scatters from the interface. In this case the potential of a three-dimensional trilayer with free-electron bands is invariant with respect to translations parallel to the interfaces. It is energetically equivalent to an aggregate of one-dimensional trilayer, the electrons in each member having the same wave-vector \( k_{||} \), parallel to the interface. Thus the energy in three dimensions is obtained by integrating the result (1.20) over the two-dimensional space of \( k_{||} \). In the limit of weak reflections and large \( t \), the result with an
Figure 1.8: The cross-section of the Fermi surface of Cu along the (110) plane passing through the origin. The solid dots indicate the reciprocal-lattice vectors. The dashed lines indicate the boundary of the first Brillouin zone. The solid arrows, respectively horizontal, oblique, and vertical, indicate the vectors $Q$ giving the oscillation period(s) for the (001), (111), and (110) orientations.

inserted temperature factor \[45,63\] for a metallic spacer is given by

$$J(t) = \left( \frac{\hbar^2 Q^2}{4\pi^2 m_e t^2} \right) \left[ \Delta R \right]^2 \sin \left( 2Q\perp t \right) \left( \frac{\zeta}{\sinh \zeta} \right), \quad (1.21)$$

where $\Delta R = R^\uparrow - R^\downarrow$, $\zeta = 2\pi k_B T m_e t/\hbar^2 Q\perp$ and $Q\perp$ is a vector spanning the non-spherical Fermi surface.

As it follows from the Eq.(1.21) the coupling decreases as the temperature increases. It should be noted that temperature dependence for a metallic and an insulating spacer are different. For detail see Refs. \[63\].

The above integration of the one-dimensional behaviour of an equation similar to Eq. (1.20) over the space $k||$ mixes different oscillation periods. However, the period $\pi/Q$ present in Eq. (1.20) survives asymptotically, because a group of waves with $k||$ near 0 gives nearly the same period. Note that the integration over parallel momentum contributes an additional power of thickness, $t$, in the decay of the envelope, and a phase shift from a sine to a cosine in the asymptotic region. An extension of this consideration gives rise to the spanning vector construction of the oscillation frequency from the shape of a realistic Fermi surface based on computed band structure of the bulk spacer material \[64, 65\]. In particular it allows one to predict the oscillation period(s) of the interlayer exchange coupling versus spacer thickness just by inspecting the bulk Fermi surface of the spacer material. Figure 1.8 shows a cross-section of the Fermi surface of Cu, indicating the stationary spanning vectors for the (001), (111), and (110) crystalline orientations \[54\]; the Fermi surfaces of Ag and Au are qualitatively similar. For the (111) orientation, a single (long) period is predicted; for the (001) orientation, both a long period and a short period are predicted; for the (110) orientation, four different periods are predicted (only one stationary spanning vector is seen in figure 1.8, the three others being located in other cross-sections of the Fermi surface).

In conclusion it should be noted that all results discussed above had been derived in assumption of infinite thickness of ferromagnetic layers ($L_A$ and $L_B \to \infty$, see Fig. 1.3). If the ferromagnetic layers are of finite thickness, reflections may usually take place at the two
interfaces bounding the ferromagnetic layers, giving rise to interferences [66], and, hence, to oscillations of the IEC versus ferromagnetic layer thickness. A more detailed discussion of this effect is given in references [63, 66]. The amplitude of the oscillations of the IEC versus ferromagnetic layer thickness is generally much smaller than the oscillations versus spacer thickness, and does not give rise to changes of sign of the IEC. From the experimental point of view, this effect was confirmed by Bloemen et al. [67] for Co/Cu/Co(001) and by Back et al. [68] for Fe/Cu/Co(001). It has also been confirmed theoretically by Nordström et al. [69], Lang et al. [70], Drchal et al. [71], and by Lee and Chang [72].

1.4 Exchange coupling in Co-based multilayer with Pt, Pd, Ru and Ir interlayers

A great variety of experimental techniques have been used to study the interlayer exchange coupling, including magnetometry, ferromagnetic resonance techniques, neutron scattering, and magnetic domain microscopy. Here we mainly focus on the simplest and most widely used magnetometry method. The magnetometry method is based on measurements of magnetoresistance, magnetization or magnetooptical Kerr effect (MOKE) loops [73]. The principle is to use an external field to bring the system from antiparallel to parallel alignment. In Fig. 1.9a we show the MOKE hysteresis loop for isolated $[\text{Co/Pt}]_5$ stack. In Fig. 1.9b the set of MOKE hysteresis loops for $[\text{Pt}(7 \, \text{Å})/\text{Co}(4 \, \text{Å})]_X/\text{Ru}(S)/[\text{Co}(4 \, \text{Å})/\text{Pd}(7 \, \text{Å})]_X$ with different Ru thickness $S$ as indicated in the plots. The sub-layer stack thickness is kept constant at $X=5$ repeats [20].

The total energy per unit volume of the simplified model for such magnetic multilayers showed schematically in Fig. 1.10a can be represented in the following form:

$$
e_{\text{tot}} = \frac{J}{lX} (m_1 \cdot m_2) - K_u \left[ (m_1 \cdot n)^2 + (m_2 \cdot n)^2 \right] - H M_s \left[ (m_1 \cdot n) + (m_2 \cdot n) \right] + 2\pi M_s^2 \left[ (m_1 \cdot n)^2 + (m_2 \cdot n)^2 \right],$$  \hspace{1cm} (1.22)
where $\mathbf{m}_i$ is a unity vector along the magnetization of $i$-th block, $J$ is the exchange constant of the antiferromagnetic interlayer coupling, $\mathbf{n}$ is a unity vector normal to the multilayer surface, $H$ is the external field perpendicular to the multilayer surface, the last term in Eq. (1.22) is the stray field energy.

We will consider the case of strong anisotropy when only collinear phases minimize the system energy. Furthermore we assume that the magnetic layers in each stack are homogeneously magnetized. In infinite layer approximation it leads to that ferromagnetic layers do not interact with other stacks due to the localization of their stray fields within the layers (see Sect. 1.5). Introducing the angles $\theta_i$ between $\mathbf{m}_i$ and $\mathbf{n}$ we describe the following four phases: ferromagnetic with $\theta_1 = 0, \theta_2 = 0$ (or $\theta_1 = \pi, \theta_2 = \pi$), and antiferromagnetic with $\theta_1 = \pi, \theta_2 = 0$ (or $\theta_1 = 0, \theta_2 = \pi$). The transition fields corresponding to the states with equal energies of FM and AF states follows directly from Eq. (1.22)

$$H_{ex} = \pm \frac{J}{M_s X t},$$

where sign “+” and “–” correspond to the $E_{\theta_1=\pi,\theta_2=\pi} = E_{\theta_1=0,\theta_2=\pi}$ and $E_{\theta_1=0,\theta_2=0} = E_{\theta_1=0,\theta_2=\pi}$ respectively. In experiment this field is usually associated with the shift of the major or minor magnetization loop (see Fig. 1.9b) and gives the simplest way to define coupling strength. In Fig. 1.10b we show the dependence of $H_{ex}$ measured for the corresponding set of MOKE loops shown in Fig. 1.9b. For the right scale of Fig. 1.10b we use equality $J = 2M_s H_{ex} X t$ with $M_s = 1420$ emu/cm$^3$ (bulk cobalt) $t=4$ Å and $X=5$.

Oscillatory coupling between Co layers across Ru spacer was first reported by S.S.P. Parkin et al. [43]. The studied multilayer structure with in plane magnetization was of the form (111)Si/Ru(100 Å)/[Co(18 Å)/Ru(S)]$_{20}$/Ru(50 Å) with Ru layer thickness $S$ varying in wide range from 3 to 38 Å. The IEC oscillation period was found of about 12-14 Å, and the antiferromagnetic coupling strength for Ru thickness 8 Å was of 5 erg/cm$^2$. This coupling strength is much larger than the ones observed in other systems. For example in multilayer composed on perpendicular [Co/Pt]$_X$ multilayer antiferromagnetically coupled via Ru or Ir layer (see Fig. 1.10b) the corresponding value for maximal coupling strength is about 0.34
Figure 1.11: \( J \) as a function of the thickness of the Ru spacer layer \( S \) for the three series of \([\text{Pt}(s_1)/\text{Co}(4 \, \text{Å})]/\text{Ru}(S)/[\text{Co}(4 \, \text{Å})/\text{Pt}(s_2)]_5\) samples: (a) \( s_1 = s_2 = 2 \, \text{Å} \) (open circles), (b) \( s_1 = 2 \, \text{Å}, \ s_2 = 11 \, \text{Å} \) (crosses), and (c) \( s_1 = s_2 = 11 \, \text{Å} \) (open squares) [75].

The same order of magnitude of IEC for \([\text{Pt}/\text{Co}]/\text{Ru}/[\text{Co}/\text{Pt}]\) systems (0.1-1 erg/cm\(^2\)) was recently reported in Refs. [74] and [75]. In Ref. [75] authors report on good agreement between experimentally measured and theoretically predicted exchange coupling (see Secs. 1.3.2 and 1.3.3). In Fig. 1.11 both the measured and theoretically calculated value of IEC are shown. As it is seen in figure the calculated peak separations agree well with the experiments within an error of ±0.04 nm and that most of the peak amplitudes from the calculations approach the experiments except that of the third peak on the curve of samples (a). It was shown in Ref. [75] that the strength of the interlayer AF coupling varies with the Pt layer thickness which may be attributed to variation of the exchange splitting and polarization in the Co layers induced by the strong hybridization between Pt 5\(d\) and Co 3\(d\) electronic states at the Pt/Co interface. Thus the interlayer coupling in the Pt/Co/Ru/Co/Pt stacks is different from that in Co/Ru/Co sandwiches.

The interfacial properties of Pt and Pd perpendicular multilayer structures are similar. There are no significant differences with respect to Pt or Pd for the AF-coupled structures as \{[\text{Pt(or Pd)}/\text{Co}]_X/\text{Ru(or Ir)}\}_N [20].

It was mentioned in literature earlier that in Co-based multilayer Pd and Pt show strong ferromagnetic coupling with no evidence for oscillatory coupling [76]. However, recently J. W. Knepper and F. Y. Yang have reported on oscillatory interlayer coupling with a ferromagnetic background as a function of the Pt thickness in \([\text{Co}(4 \, \text{Å})/\text{Pt}(s)]_X\) perpendicular multilayers with repetition \(X\) from 2 to 30 and Pt thicknesses \(s\) from 3 to 79 Å [77]. First antiferromagnetic coupling in \([\text{Co}(4 \, \text{Å})/\text{Pt}(s)]/[\text{Co}(4 \, \text{Å})/\text{Pt}(6 \, \text{Å})]_2\) multilayer with perpendicular anisotropy have been reported by X.-X. Li et al. in [78] for Pt layer thickness \(s \geq 22 \, \text{Å}\). Then in Ref. [79] Z. Y. Liu et al. have presented direct evidence for the existence of antiferromagnetic interlayer coupling between the Co layers in \([\text{Co}(4 \, \text{Å})/\text{Pt}(s)]_3\) multilayer through the Pt layers thicker than 24 Å. In the same year in Ref. [80] Z. Y. Liu et al. reported that antiferromagnetic interlayer coupling occurs between the soft single Co layer and the hard \([\text{Co}(4 \, \text{Å})/\text{Pt}(6 \, \text{Å})]_X\) multilayer across the 36 Å Pt spacer, and the coupling strength oscillates...
as a function of the repetition number $X$. As it shown in Fig. 1.12 the value of antiferromagnetic coupling strength across Pt layer is always two orders of magnitude less than the corresponding antiferromagnetic coupling for Ru interlayer. No evidence of antiferromagnetic coupling in Co-based multilayer across Pd layer has been reported.

### 1.5 The Origin of Domains

In this subsection we give qualitative arguments supporting the existence of magnetic domains. It turns out to be impossible to assign a single origin to domain structures in all kinds of materials. Somehow the non-local energy terms, above all the stray field energy, are responsible for the development of domains. But the arguments for the occurrence of domains differ considerably depending on the magnitude of the anisotropies and on the shape and size of the samples [2]. Here we limit the discussion to thermodynamically stable domains in extended uniaxial ferromagnetic films with strong perpendicular anisotropy.

First let us consider homogeneous magnetized film in the absence of external magnetic field (Fig. 1.2, a). Outside the film the magnetic field $H_d$ is zero analogous to a uniformly charged capacitor in electrostatics. Inside the magnetic thin film there is a uniform demagnetization field $H_d$ and its energy density $e_d$, per unit area:

$$H_d = -4\pi M_s, \quad e_d = \frac{1}{2} H_d \cdot M_t = 2\pi M_s^2 t. \quad (1.24)$$
As this is a particularly unfavourable situation, the quantity $2\pi M_s^2$ is a measure for the maximum volume energy densities which may be connected with stray fields. Because the contributions from anisotropy energy and exchange energy are equal zero, there is only one non-zero contribution to the total energy of the system. It is the magnetostatic energy. To lower this energy, the system can form perpendicular domains of average width $d$ (Fig. 1.13, b) at the expense of introducing domain wall energy. In the vicinity of these domain walls we now observe external stray fields (Fig. 1.13, b). The total magnetic flux depends on density of magnetic charges and should be constant. Therefore, internal stray field and consequently stray field energy Eq. (1.12) is reduced. A periodic parallel stripe domain pattern is determined by the balance between the stray field energy reduction versus the domain wall energy increase.

It is useful for further discussion to introduce an abbreviation

$$K_d = 2\pi M_s^2.$$  \hspace{1cm} (1.25)

For a plate uniformly magnetized perpendicular to its surface the stray field energy density is equal to $K_d$. The quantity $K_d$ is a measure for the maximum stray field energy densities. The material parameter $K_d$, usually called *stray field energy coefficient*, has the order of magnitude varying from zero to $3 \times 10^7$ erg/cm$^3$ ($3 \times 10^6$ J/m$^3$) [2].

### 1.6 Domain Walls

The calculation of domain wall structure is one of the most important contribution of micromagnetics to the analysis of magnetic domains. Perhaps the simplest and at the same time the most fundamental magnetic structures in micromagnetism are the $180^\circ$ domain walls. These domain walls describe the magnetic structure in the transition region between two extended regions which are magnetized homogeneously and in opposite direction (see Fig. 1.14). In the limit of domain theory, where the internal structure of domain walls is neglected, the domain wall is the line (in two dimensions) or surface (in the three-dimensional case) separating the mutually antiparallel domains in uniaxial ferromagnets. In the framework of micromagnetism the continuous transition of the magnetization is studied. Accordingly, the domain wall can there be defined as the line or the surface on which the magnetization is oriented perpendic-
Figure 1.14: (Color) The rotation of the magnetization vector from one domain through a 180° wall to the other domain in an infinite uniaxial material. Two alternate rotation modes are shown: the optimum mode, which is called the Bloch wall, as compared to the Néel wall, which is less favourable here but can be preferred in thin films and in applied fields. For both modes the opposite rotation is equally possible.

ular to both domains, i.e., the region in which the magnetization has performed half of the reorientation between one domain direction and the other. The classical case, which will be discussed in the following, is that the domain wall is oriented parallel to the magnetization direction. The famous Bloch wall and the Néel wall are both of this type. A different category of domain wall which has been discussed intensively in the last years are various forms of head-to-head domain walls, where the magnetization in the domains is perpendicular to the domain wall. For simplicity it may be assumed that the magnetization does not display any variation along the domain wall. The orientation of the magnetization then only depends on the distance from the domain wall, such that the description of the domain wall structure is a one-dimensional problem. There are two fundamentally different possibilities for the magnetization to rotate continuously from the direction of one domain to the other, known as Bloch wall and Néel wall. These one-dimensional domain walls are the simplest domain wall types. Needless to say that they only represent idealized situations, and that in reality a much larger spectrum of domain wall types can be found. In particular, two-dimensional domain walls like crosstie walls, asymmetric Bloch walls or asymmetric Néel walls are important and fascinatingly complex “hybrid” domain walls, which display combinations of both fundamental types: the Bloch wall and the Néel wall.

1.6.1 Bloch wall

The 180-degree Bloch wall describes the rotation of the magnetization in a small region between domains with antiparallel magnetization. Bulk ferromagnetic material with uniaxial anisotropy is assumed. Effects connected with the surface of the sample are not considered and the magnetization in the domains is aligned with the easy axis of the anisotropy. In the case of a Bloch wall the magnetization always remains perpendicular to the wall normal, around which it rotates by 180° in the transition region.

To describe this situation it is useful to select a specific coordinate frame in which the \( x \)-component of the magnetization is the component perpendicular to the domain wall. In
the case of a Bloch wall, this $m_x$, component is everywhere equal to zero. The domain wall profile ($m_y(x), m_z(x)$) only depends on the $x$ coordinate and can be conveniently described by polar coordinates in the form

$$m_z(x) = \cos \vartheta(x), \quad m_y(x) = \sin \vartheta(x). \quad (1.26)$$

With the given boundary conditions

$$\vartheta(-\infty) = 0, \quad \vartheta(\infty) = \pi \quad (1.27)$$

and the selected sense of rotation of the magnetization, the static magnetic structure describing the domain wall profile $\vartheta(x)$ can be obtained from energy minimization. The involved energy terms are the exchange energy and the anisotropy energy. These are competing interactions in the given situation. The exchange energy is minimized if the domain wall is wide, i.e., if the transition occurs very smoothly over a large distance, since then the inhomogeneities ($\nabla m_i)^2$ are small. On the other hand, the anisotropy energy tends to reduce the width of the domain wall as much as possible in order to align the magnetization with the easy axes. The result of this competition is obtained by minimizing the domain wall energy $\gamma_B$:

$$\gamma_B = \int_{-\infty}^{\infty} \left[ K_u \sin^2 \vartheta + A \left( \frac{d\vartheta}{dx} \right)^2 \right] dx. \quad (1.28)$$

The minimization $\delta \gamma_B = 0$ can be performed analytically and yields

$$\cos \vartheta = \tanh \left( x / \sqrt{A/K_u} \right). \quad (1.29)$$

The profile of the magnetization of this wall type is sketched in Fig. 1.15. A frequent definition of the domain wall width $\delta_B$ is given by the distance between the points at which the tangent at $x = 0$ crosses the $\vartheta = 0$ and the $\vartheta = \pi$ lines. With this definition due to
Lilley [81], the domain wall width of a Bloch wall is
\[
\delta_B = \pi \sqrt{A/K_u}.
\]  
(1.30)
The energy density \(\delta_B\) of a Bloch wall (defined as the energy per unit area of the domain wall) results to be
\[
\gamma_B = 4 \sqrt{AK_u}.
\]  
(1.31)
An important property of the Bloch wall is that the magnetization distribution is free of divergence. Therefore, no bulk charges \(\rho = -\nabla M\) are formed, which would be sources of magnetostatic stray fields. Since, by definition of the problem, surface charges are neglected, there are no sources of the magnetostatic field present and it is thus legitimate to neglect the dipolar energy term in the energy minimization. The Bloch wall has been derived under the assumption of bulk ferromagnetic material with uniaxial anisotropy. The term “bulk” means that the sample is very thick, such that surface effects can be neglected. In contrast to this, the Néel wall is the classical domain wall type occurring in thin, soft magnetic films.

1.6.2 Néel wall

The difference of the domain wall type between the thin-film case and the bulk case is due to magnetostatic effects. In bulk samples (i.e., thick samples of typically about 100 nm thickness and above) it may be quite safely neglected that a small amount of surface charges occurs in a Bloch-type domain wall transition as the magnetization rotates by 180°. But in thin films, the relative impact of the magnetostatic energy connected with the field created by these charges is much larger. Néel has demonstrated [32] that in these cases it is energetically more favourable for the magnetic system to perform the change of the magnetization direction by a rotation in the film plane. This transition is sketched in Fig. 1.14.

The Néel wall profile can be calculated by the minimization of a one-dimensional energy functional, analogous to the case of the Bloch wall described before. In the case of Néel walls the competing interactions are given by the magnetostatic energy and the exchange energy. The tendency of the magnetostatics to keep the domain wall as small as possible is balanced by the tendency of the exchange term to avoid strong inhomogeneities of the magnetization. The magnetostatic field in the case of a Néel wall is calculated using Eq. (1.10). The one-dimensional energy functional to be minimized is then
\[
\gamma_N = \int_{-\infty}^{\infty} \left[ 2\pi M_s^2 \cos^2 \vartheta(x) + A \left( \frac{d\vartheta}{dx} \right)^2 \right] dx.
\]  
(1.32)
The result of this minimization yields the profile of the Néel domain wall
\[
\cos \vartheta(x) = \tanh \left( x/\sqrt{A/2\pi M_s^2} \right),
\]  
(1.33)
which has a form that is very similar to that previously derived for the Bloch wall. The domain wall width of a Néel wall is accordingly
\[
\delta_N = \pi \sqrt{A/2\pi M_s^2}.
\]  
(1.34)
and the energy density of the Néel wall is

$$\gamma_N = 4\sqrt{A2\pi M_s^2}. \quad (1.35)$$

Evidently, the role played by the stray field constant $K_d$ for Néel walls is analogous to that of the anisotropy constant $K_u$ in the case of Bloch walls. The ratio of anisotropy constant $K_u$ to stray field constant $K_d$ is the definition of the quality factor $Q$ that indicates the magnetic hardness. In this context it is not surprising that the quantities $K_u$ and $K_d$ appear in the comparison between Bloch an Néel wall, since the Bloch wall has been calculated for hard magnetic materials with uniaxial anisotropy while the Néel wall is derived for ideally soft materials ($K_u = 0$). However, the selection between Néel and Bloch type domain wall is not primarily given by the magnetic hardness, but rather by the film thickness. In thick samples Bloch walls are favoured, in thin films Néel walls. Of course the terms “thin” and “thick” are only relative terms which require a comparison with a well-defined length scale to become meaningful. As far as the distinction between Néel and Bloch wall type is concerned, it can roughly be estimated that a film can be considered to be thin if the domain wall width is larger than the film thickness, whereas in thick films it is the opposite.

It should be noted that Néel type domain walls are magnetically charged. Contrary to the Bloch wall, the magnetic structure of Néel walls displays a non-vanishing divergence, so that magnetic volume charges are connected with Néel walls. The magnetic fields created by these volume charges are relatively small as long as the film thickness is low.

For the typical cobalt exchange constant value $A = 2 \times 10^{-6}$ erg/cm (see Sect. 1.2.1) and effective value of uniaxial anisotropy $K_u = 6.5 \times 10^6$ erg/cm$^3$ (see Sect. 1.2.2) and saturation magnetization for bulk cobalt $M_s = 1400$ emu/cm$^3$ the width of Bloch-wall (Eq. 1.30) and the Bloch-wall energy density (Eq. 1.31) are respectively $\delta_B \approx 17 \text{ nm}$ and $\gamma_B \approx 14 \text{ erg/cm}^2$.

The width of Néel-wall (see Eq. 1.34) and the Néel-wall energy density (see Eq. 1.35) are respectively $\delta_N \approx 13 \text{ nm}$ and $\gamma_N \approx 20 \text{ erg/cm}^2$.

### 1.6.3 Domain walls in magnetic single layers and multilayers with perpendicular anisotropy

The one dimensional micromagnetic problems described in Sects. 1.6.1 and 1.6.2 are the simplest way to understand how the balance between main energy terms stabilize the equilibrium magnetization distribution in domain walls. Now let us consider more complicated case of thin ferromagnetic layer with perpendicular anisotropy. As it was shown in Sect. 1.5 stray field effects in such a layer lead to the formation of a regular domain structure. In Fig. 1.16 we show the domains connected with stray fields. In the walls the magnetization lies parallel to the surface along different directions. This is the typical situation of a bubble film or a magnetic layer of perpendicular anisotropy.

The basic features of walls in perpendicular films were analysed by Slonczewski [82] and Hubert [83]. It was shown that the wall is a regular Bloch wall in the centre of the film. Towards the surfaces the stray field from the domains acts on the wall twisting the magnetization towards a Neel wall.

The approach of Slonczewski and Hubert is based on following simplified model. The horizontal stray field caused by the domains is almost singularly high right at the surface. They considered a periodic arrangement of bar-shaped perpendicular domains with infinitely thin
domain walls and an infinite extension in the $y$ direction (Fig. 1.16). Then the $x$ component of the stray field in the wall centre plane can be expressed analytically. This approach allows one to reveal main qualitative features of domain walls in perpendicular magnetic layer. However, realistic model for magnetization distribution in two-dimension domain wall in perpendicular layers in general does not have analytical solution. Due to this fact numerical micromagnetic simulations are widely used to determine the detailed magnetization distribution in thin films. The detail theoretical study of magnetization distribution in thin magnetic layers with out of plane magnetization can be found in papers by M. Labrune et al. (see e.g. Refs. [84,85]).

The first qualitative analysis of magnetic structure of domains and domain walls in magnetic multilayers have been done in Ref. [86]. The magnetic microstructure of a Co/Pt multilayer was simulated by solving the Landau-Lifshitz-Gilbert equations adapted for a multilayer structure. Fig. 1.17 shows the typical magnetisation distribution generated in the simulation with the use of finite element method (the data are taken from Ref. [87]). The general features of the micromagnetic structure in Fig. 1.17 can be summarized as follows: within domains, the magnetisation direction is not constant throughout the magnetic layer thickness and the walls themselves exhibit a strong two dimensional character.

The domains in Co/Pt multilayers can be imaged in a scanning transmission electron microscope by using the modified differential phase contrast mode of Lorentz electron microscopy. The predicted wall structure is consistent with the experimental observations in terms of both scale and contrast [86].

Such a wall would never occur in a very thin continuous film because the increment of the exchange energy due to abruptly changing the sense of rotation of the in-plane component at the centre of the film would be too high. Of course, in thicker films it is not unusual to find that at the surfaces the wall shows Néel-like behaviour with opposite polarities observed at top and bottom. However, in these cases there is always an intermediate Bloch-like section through the centre of the film leading to the well-known vortex walls [88,89]. As the anisotropy energy is probably similar, whether we have Bloch- or Néel-like behaviour, it is not surprising that the micromagnetic calculation, which imposes no conditions about the sense in which the magnetization should rotate, finds a lower energy state than would be possible with a simple one-dimensional variation. In the case of ferromagnetic IEC a large IEC would inhibit
Figure 1.17: 3D magnetization profiles of a) a single layer 42 nm thick and b) a bilayer (Co 20 nm/spacer 2.1 nm/Co 20 nm). The scale along the OX axis is normalized to one period (the size of the slabs are assumed infinite along OZ) [87].

the flux closure reducing the wall energy. It was found in Ref. [90] that the effective wall energy increases rapidly between the values 0 and 0.1 for the ratio of the IEC $J$ to the bulk exchange coupling $A$, and remains constant thereafter.

The effective perpendicular anisotropy (see Sect. 1.2.2) also affects magnetization distribution in multilayers. In Fig. 1.18 we show the magnetization pattern cross section of the $[\text{Co}(1.2 \text{ nm})/\text{Pt}(2.1 \text{ nm})]_n$ multilayer stack composed of $n=10$ magnetic layers over one period of the stripe for two different surface anisotropy constants: $K_s=0.3$ and 0.6 erg/cm$^2$ [mJ/m$^2$], respectively (the data are taken from Ref. [91]). Other material parameters of the system are saturation magnetization $M_s=1430$ erg/cm$^3$ [kA/m], exchange stiffness parameter $A = 1.8 \times 10^{-6}$ erg/cm [$1.8 \times 10^{-11}$ J/m], bulk anisotropy $K_v = 4.8 \times 10^6$ erg/cm$^3$ [4.8$\times$10$^5$ J/m$^3$]. For higher surface anisotropy, i.e. $K_s=0.6$ erg/cm$^2$, corresponding to an effective quality factor larger than one, all domains are mainly oriented up and down as depicted in Fig. 1.18b. On the other hand, a lower interface anisotropy constant allows a pronounced magnetization oscillation (see Fig. 1.18a) leading to a practically stray field free pattern. Further decrease of $K_s$ (for this system $K_s \leq 0.28$ erg/m$^2$) leads to a transition towards a uniform in-plane magnetization pattern.

Vortex-like domain walls discussed here have energy density less than Neel or Bloch wall. Therefore, expected value of domain walls energy and domain sizes presented in previous subsection can be viewed as upper estimate for these quantities.

Figure 1.18: Magnetization pattern of the $[\text{Co}(1.2 \text{ nm})/\text{Pt}(2.1 \text{ nm})]_{10}$ multilayer cross section for: a) $K_s=0.3$ erg/cm$^2$ and period of stripe domains 160 nm; b) $K_s=0.6$ erg/cm$^2$ and period of stripe domains 170 nm [91].
1 Phenomenological Theory of Multidomain States in Thin Ferromagnetic Layers

1.7 Stripe and Bubble Domains in A Ferromagnetic Single Layer

This final introduction section is devoted to a model of periodic stripe and bubble domains originally put forth by Kittel [1] and further developed by Málrek and Kamberský [21] as well as Kooy and Enz [22].

1.7.1 Stripe Domains

Let us consider a domain structure in an infinite plate of thickness $t$ with stripe domains schematically showed in Fig. 1.19. Note, that the term “stripe domains” is also commonly used to denote multidomain patterns consisting of stripes with weakly undulating magnetization which, however, stays predominantly in the layer plane [2]. On the other hand, the terms band domains or strong stripes are used to describe structures of homogeneous domains with perpendicular magnetization that alternates between up and down direction. We assume that [22]:

(i) The material is uniaxial with the preferential direction of the magnetization normal to the surface of the plate.

(ii) The plate has the uniform thickness $t$ and its lateral dimensions in the $x$ and $y$ directions are infinite.

(iii) The anisotropy field is strong, so that closure domains do not exist. Under the strong anisotropy we assume a high quality factor, i.e. the ratio $Q = K_u/K_d \gg 1$. However, we will take into account that under the influence of the demagnetizing field the direction of the magnetization may depart slightly from the $z$-axis ($\mu$-effect).

(iv) The external magnetic field $H$ is applied parallel to the surface normal.

(v) The thickness of the plate is assumed to be small enough so that the domain structure can be considered to consist of straight domains with the demagnetization as well as the $180^\circ$ Bloch walls perpendicular to the surface.

The total energy of this system consists of contributions from domain wall energy $E_w$, 

Figure 1.19: (Color) Schematic representation of stripe domain structure in a ferromagnetic monolayer. The width of domain magnetized in field and opposite direction marked as $d_+$ and $d_-$ respectively. The period of stripe domain $D = d_+ + d_-$. 

35
Zeeman energy $E_H$, and magnetostatic energy $E_d$. Let us consider each of them sequentially.

The energy of an isolated domain wall can be written as

$$E_w = \int l \gamma_w dl$$  \hspace{1cm} (1.36)

where $\gamma_w$ is a surface energy density of a domain wall and $l$ is a total length of the wall along the domain wall outline. For an infinite plate this energy goes to infinity but the energy density per unit surface area of the layer remains finite. The number of domain walls per unit surface area is $n = 2/D$, where the factor two appears because one period has two domain walls. Then, the energy density of the domain walls:

$$e_w = \frac{2t\gamma_w}{D}. \hspace{1cm} (1.37)$$

According to Eq.(1.9) Zeeman energy density can be written as

$$e_H = H (M_s d_+ - M_s d_-) t/D. \hspace{1cm} (1.38)$$

The sign “−” in the second term appears because the magnetization in the minority phase domain of width $d_-$ is opposite to the applied field.

Now let us consider the magnetostatic energy contribution. Since the magnetization in each domain is assumed to be homogeneous only surface charges $\sigma$ at the surfaces of the magnetic layer are present ($\lambda = 0$) and therefore the Poisson equation (1.15) becomes Laplace equation

$$\Delta \Phi_d = 0 \hspace{1cm} (1.39)$$

with corresponding boundary conditions on upper and lower surfaces respectively

$$\left. \frac{\partial \Phi^i}{\partial z} \right|_{z=0} - \left. \frac{\partial \Phi^{+e}}{\partial z} \right|_{z=0} = +4\pi\sigma$$

and

$$\left. \frac{\partial \Phi^i}{\partial z} \right|_{z=-t} - \left. \frac{\partial \Phi^{-e}}{\partial z} \right|_{z=-t} = -4\pi\sigma$$

where $\sigma = \sigma_s(x)$ is the density of surface magnetostatic charge on upper surface (sign “+”) and lower surface (sign “−”), $\Phi^i$ and $\Phi^e$ are the internal scalar potential within the layer and external potential outside the layer, the superscripts “+e” and “−e” correspond to the half-spaces above the layer and below the layer respectively. The function for the distribution of surface charge on the upper surface of the layer $\sigma_s(x)$ for the regular stripe domains can be written as Fourier series expansion:

$$\sigma_s(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \left(\frac{2\pi nx}{D}\right) \hspace{1cm} (1.40)$$
where

\[ a_0 = \frac{2}{D} \int_0^D \sigma_s \, dx = \frac{2M_s}{D} (2d_+ - D), \]

\[ a_n = \frac{2}{D} \int_0^D \sigma_s \cos (2\pi nx/D) \, dx = \frac{4M_s}{\pi n} \left[ \sin (\pi nd_-/D) \cos (\pi n) \right], \]

and \( \sigma_s \) is defined as:

\[ \sigma_s = \begin{cases} +M_s, & -d_+/2 + nD < x < d_+/2 + nD, \\ -M_s, & d_+/2 + nD < x < -d_+/2 + (n + 1)D. \end{cases} \]  

(1.41)

Taking into account that \( \cos (\pi n) = (-1)^n \) and \( \sin (\pi nd_-/D) = (-1)^{n+1} \sin (\pi nd_+/D) \), it is easy to show that the coefficients in the Fourier series finally can be written as

\[ a_0 = \frac{2M_s (d_+ - d_-)}{D}, \quad a_n = \frac{4M_s}{\pi n} \sin (\pi nd_+/D). \]  

(1.42)

Under the condition of continuity of magnetostatic potential on the upper and lower surfaces \( (\Phi^i = \Phi^{\pm e}) \) the solution of Laplace’s equation (1.39) for the stripe domain pattern can be represented in the following form:

\[ \Phi_s(x, z) = \begin{cases} \Phi^+_s(x, z) = \phi_s(x, z) - \phi_s(x, z + t), & z > 0 \\ \Phi^-_s(x, z) = \phi_s(x, -z) - \phi_s(x, z + t), & -t < z < 0 \\ \Phi^{\pm e}_s(x, z) = \phi_s(x, -z) - \phi_s(x, -z - t), & z < -t \end{cases} \]  

(1.43)

where

\[ \phi_s(x, z) = -2\pi M_sz \frac{2d_+ - D}{D} + M_s \sum_{n=1}^{\infty} \frac{4D}{\pi n^2} \sin \left( \frac{\pi nd_-}{D} \right) \cos \left( \frac{2\pi nx}{D} \right) \exp \left( -\frac{2\pi nz}{D} \right) \]  

(1.44)

is a function which has physical meaning of scalar potential resulting from the magnetic charges on the one of the surfaces of the layer. Strictly speaking this definition is not correct as magnetic charges never appear isolated (see Sect. 1.2.4) and only functions \( \Phi^{\pm e}_s \) have rigorous physical meaning. Nevertheless, we can use this definition under the condition that we always take into account the balance between positive and negative charges.

In accordance with the above and by using Eq. (1.16) the magnetostatic self energy density of a single magnetic layer per unit area can be expressed as

\[ e_{m, s} = \frac{1}{2} \int_0^D (\sigma_s(x)\phi_s(x, 0) - \sigma_s(x)\phi_s(x, t) - \sigma_s(x)\phi_s(x, t) + \sigma_s(x)\phi_s(x, 0)) \, dx. \]  

(1.45)

The first term in Eq. (1.45) is the interaction between upper surface charges with their own potential, the second term is the interaction between upper surface charges with the potential from charges on the opposite (lower) surface. The third and fourth terms are the same as first and second but for the charges on lower surface. After the substitution Eqs. (1.40) and
(1.44) into Eq. (1.45) and integration the magnetostatic self energy has the form:

\[
e_{m, s} = 2\pi M_s^2 t \left( m^2 + \frac{4}{\pi^2 p} \sum_{n=1}^{\infty} \frac{1}{n^3} \left\{ 1 - (-1)^n \cos(\pi nm) \right\} [1 - \exp(-np)] \right),
\]

where \( m = (d_+ - d_-)/D \) is reduced magnetization, \( p = 2\pi t/D \) is a parameter inversely proportional to the reduced period. In Eq.(1.46) we used the equalities

\[
\sin^2(\pi nd_+/D) = \left\{ 1 - (-1)^n \cos(\pi nm) \right\}/2
\]

and

\[
\int_{0}^{D} \cos(2\pi kx) \cos(2\pi nx) \, dx = \begin{cases} D/2, & k = n \\ 0, & k \neq n \end{cases}.
\]

Thereby, the total energy density of the stripe domain pattern in ferromagnetic monolayer reduced to \( 2\pi M_s^2 t \) can be written as

\[
e_{\text{tot}} = m^2 + \frac{4}{\pi^2 p} \sum_{n=1}^{\infty} \frac{1}{n^3} \left\{ 1 - (-1)^n \cos(\pi nm) \right\} [1 - \exp(-np)] + \frac{2l_c p}{\pi t} - 2hm,
\]

where \( l_c = \gamma w/(4\pi M_s^2) \) is the characteristic length, \( h = H/(4\pi M_s) \) is the reduced external field.

In common magnetic systems with characteristic sizes far beyond the nanoscale range the equilibrium domain sizes are usually much smaller than the individual layer thicknesses, \( p \gg 1 \). Numerous observations indicate that, as soon as domain sizes approaches the layer thickness, coercitivity suppresses the formation of regular multidomain patterns [2]. This establishes a natural limit for domain sizes in classical systems. For \( D \ll t \) there is no effective dipole interaction between opposite surfaces. Hence, in Eq.(1.46) we can neglect the exponent term in magnetostatic energy \( (\exp(-np) \ll 1) \), and the total energy of the multilayer Eq.(1.47) at zero field \( (m = 0) \) is reduced to

\[
e_{\text{tot}} = \frac{4l_c}{D} + \frac{2D}{\pi^3 t} \sum_{n=1}^{\infty} \frac{1}{n^3},
\]

and minimization with respect to domain period \( D \) immediately yields the dependence of equilibrium domain period on the thickness of magnetic layer [1]:

\[
D = \sqrt{\frac{2\pi^2 tl_c}{\zeta}},
\]

where \( \zeta = \sum_{n=1}^{\infty} n^{-3} \approx 1.0518 \). Equation (1.49) is valid only for a thick magnetic layer. However, there is a lot of evidence of existence of regular multidomain patterns in perpendicular polarized nanoscale films and multilayers with high quality factor \( Q \) where the period of domain structure is of the same order as their thickness or even exceeds it (see e.g. [5,7,11]).
such systems dipole interactions between different surfaces have a sizable effect. Mathematically, this is seen from slowly converging sums of interaction terms between poles far apart and on different internal surfaces. This means that for large stripes \((D \geq t)\) the magnetostatic energy \(E_s\) should be calculated rigorously without truncation of dipole sums \([2, 92]\). For such structures, numerical evaluation becomes arduous, and sharpened analytical methods are required. Several methods have been proposed to overcome the slow convergence in \(E_s\) \([92–94]\).

We use a method first introduced in Ref. \([92]\) and reduce the infinite sums in (1.46) into finite integrals (for detail see \([92]\)). In particular, for the magnetostatic energy Eq.(1.46) we can write:

\[
m^2 + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^3} \{1 - (-1)^n \cos(\pi nm)\} [1 - \exp(-np)] = \\
1 - \frac{2p}{\pi^2} \int_0^1 (1 - \xi) \ln \left[ 1 + \frac{\cos^2(\pi m/2)}{\sinh^2(p\xi/2)} \right] d\xi. \quad (1.50)
\]

The reduction of the stray field energy (1.46) into an integral on the interval \([0, 1]\) (1.50) is of fundamental importance for further calculations. It eliminates numerous difficulties arising due to slow convergence of the infinite sums and allows effectively to investigate the system energy with numerical and analytical methods. It will be shown in following chapters that similar integral transformations can be applied to different types of two-dimensional multidomain arrays, see Refs. \([I-VI]\).

Thereby, taking into account Eq. (1.50) the total energy of the system can be written as:

\[
e_{tot} = 1 - \frac{2p}{\pi^2} \int_0^1 (1 - \xi) \ln \left[ 1 + \frac{\cos^2(\pi m/2)}{\sinh^2(p\xi/2)} \right] d\xi + \frac{2l_c p}{\pi t} - \frac{2h_m t}{D}. \quad (1.51)
\]

After simple transformation the equations, \(\partial E_{tot}/\partial m = 0\) and \(\partial E_{tot}/\partial p = 0\), for the equilibrium values of the parameters \(m\) and \(p\) assume the form:

\[
\int_0^1 \ln \left[ 1 + \frac{\cos^2(\pi m/2)}{\sinh^2(p\xi/2)} \right] d\xi = \frac{\pi l_c}{t} \quad (1.52)
\]

\[
p\sin(\pi m) \int_0^1 \frac{(1 - \xi)d\xi}{\sinh^2(p\xi/2) + \cos^2(\pi m/2)} = \frac{H}{2M_s}. \quad (1.53)
\]

At zero field \((m = 0)\) Eq. (1.52)

\[
-2 \int_0^1 \xi \ln[\tanh(\xi p/2)] d\xi = \frac{\pi l_c}{t} \quad (1.54)
\]

yields the solutions for the equilibrium period \(D_0(t)\) (Fig. 1.20, solid line). The minimum of the function \(D_0(t)\) is derived from the equation

\[
\ln[\tanh(\xi p/2)] = \int_0^1 \xi \ln[\tanh(\xi p/2)] d\xi \quad (1.55)
\]
Figure 1.20: (Color) The equilibrium reduced stripe period $D/l_c$ as a function of the reduced layer thickness $t/l_c$ in a single ferromagnetic layer. Thick line indicates rigorous results, and dashed lines the solutions for Model I, Model II and Kittel approach for thick films. Inset shows relative differences between the rigorous stripe period solutions ($D_0$) and those for Models I and II as functions of $D_0/t$.

which has the solution $p^* = 1.595$ or $D^*/t^* = 3.939$. Because Eq. (1.54) includes no material parameters the value for reduced thickness and period at this minimum, $t^*/l_c = 0.96067$ and $D^*/l_c = 16.3136$, have a universal character.

Figure 1.21 shows the experimental plot of the domain size $D_0/2$ versus the thickness of magnetic layer in comparison to the results of calculations using formula (1.54). The calculations have been done by using the mean square value of the characteristic length $l_c$ derived straight from Eq. (1.54) for experimental data ($t$ and $D$) from Ref. [5] (see the inset in Fig. 1.21).

The nonmonotonic behaviour of $D(t)$ reflects the antagonistic role of magnetic charges in the formation of the equilibrium stripes. In the case of small domains $D \ll t$, which is typical for classical systems, the dipole interaction between different surfaces of the layer is negligibly small, and only the interaction between charges on the same surface give a contribution to the stray field energy. For stripes with sizes $D \geq t$ the interaction between charges from different surfaces becomes a noticeable effect and counteracts the interactions between charges on the same surface. This can be understood as a screening effect. As the layer thickness decreases this screening effect becomes stronger and suppresses the stray field energy. As a result, for $t < t^*$ the expansion of the domains decreases both the sum of domain wall energies and the stray field energy, and the domain period increases exponentially with decreasing layer thickness. For large stripes ($D \geq t$) the expansion of the integrals in Eqs. (1.52) and (1.53) allows to simplify the problem [92]. Here we present two simplified models describing the stripe phase in the limit of large domains.

**Model I.** For $D \geq t$ the function $\sinh(p\xi/2)$ in Eqs. (1.52), (1.53) can be replaced by its arguments. After that the integrals can be evaluated in terms of elementary functions. By
introducing a new variable $b = 2 \cos \left( \frac{\pi m}{2} / p \right)$ we reduce the system energy into the form

$$e_{\text{tot}}(b, m) = V(b) \cos \left( \frac{\pi m}{2} \right) - \frac{H_m}{2\pi M},$$

(1.56)

$$V(b) = -\frac{2}{\pi^2 b} \left[ 2\pi l_c/t + 4b \arctan(1/b) - (b^2 - 1) \ln(1 + b^2) + b^2 \ln(b^2) \right].$$

(1.57)

Minimization of (1.56) with respect to $b$ and $m$ yields the following equations

$$(1 + b^2) \ln(1 + b^2) - b^2 \ln(b^2) = \frac{2\pi l_c}{t},$$

(1.58)

$$H_{s0} = 4M_s \left[ 2 \arctan \left( \frac{1}{b} \right) - b \ln \left( 1 + \frac{1}{b^2} \right) \right],$$

(1.59)

$$m = \left( \frac{2}{\pi} \right) \arcsin(H/H_{s0}).$$

(1.60)

After some algebra, the solutions for stripes can be derived in analytical form

$$D(H) = \pi tb / \sqrt{1 - (H/H_{s0})^2}, \quad d_{\pm} = (D/\pi) \arccos(\mp H/H_{s0}),$$

(1.61)

where the functional relation between the characteristic field $H_{s0}$ and the material parameter $l_c/t$ is established via parametric equations (1.58), (1.59). According to (1.61) at zero field $D(0) = D_0 = \pi tb$, thus, the ratio $D_0/d_+ = \pi$. It means that, at the transition field, the
domain of the minority phase becomes approximately six times narrower than the domain size at zero field ($D_0/2$). Within this approximation the equilibrium magnetization in the stripe phase equals

$\langle M \rangle = M_{\perp} m = (2M_{\perp}/\pi) \arccos(H/H_{s0}).$  \hfill (1.62)

The magnetic susceptibility is

$\chi(H) = \frac{d \langle M \rangle}{dH} = M_s \frac{dm}{dH} = \frac{4M_s}{\pi \sqrt{H_{s0}^2 - H^2}}.$ \hfill (1.63)

Particularly, the initial susceptibility $\chi(0) = 4M_s/H_{s0}$, and near the transition field $\chi = \chi(0)/\sqrt{2(1 - H/H_{s0})}$.

**Model II.** For large $b$ further simplifications of stripe solutions can be performed. Expanding Eqs. (1.58), (1.59) with respect to a small parameter $1/b$ yields analytical solutions

$d^* = b = \exp(\pi l_c/t - 1/2),$  
$H_{s0}^* = 2M_s \exp(-\pi l_c/t + 1/2) \ll M_s,$  
$D(H) = \frac{\pi t \exp(\pi l_c/t - 1/2)}{\sqrt{1 - (H/H_{s0}^*)^2}}.$  \hfill (1.64)

As it is shown in the inset of Fig. 1.20 a relative error which exhibit both Models I and II in stripe period solutions is less than 2% for the reduced thickness $t/l_c \leq 2$ ($D_0/t \geq 10$). Model I is always close to the rigorous solution $D_0$. However in the limit of large domains (thin layer) both Models show good agreement with the rigorous solution.

### 1.7.2 Bubble Domains

Magnetic bubbles are cylindrical domains which can occur in thin films of uniaxial magnetic materials. In Fig. 1.22 we have drawn such a magnetic bubble in a thin single-layer, situated in a magnetic field $H$. The axis of easy magnetization and the field $H$, are both normal to the layer; we shall define this normal to the sheet as the $z$-direction.

The Kooy and Enz theory, presented in previous section was incomplete insofar as it examined only one possible configuration, that of parallel stripe domains. In particular near saturation a bubble lattice has a smaller total energy than the stripe pattern, while an individual bubble is stable in a larger field range beyond and below the saturation field of the equilibrium bubble lattice.

The interaction between bubbles arranged in a hexagonal lattice in a single magnetic layer first has been calculated in Refs. [95, 96] and experimentally verified in Ref. [97]. It was found that for a fixed value of the bias field $H$ the radius of a bubble in a lattice is smaller than the radius of an isolated bubble, because, within the film, the $z$-component of the stray field of a bubble has the same direction as $H$. The presence of the surrounding bubbles increases the effective bias field. In the region outside the layer just above a bubble, the $z$-component of the stray field is antiparallel to $H$. A bubble located next to the given bubble is subjected to a total field which is lower than $H$. In the following we consider in detail
isolated bubble domains only, while bubble lattices are reviewed without proof. For details see the textbooks [98–100].

The static stability of a single bubble in an infinite plate is the elementary task of bubble theory and first has been discussed in detail in Refs. [22, 101]. Similar to the stripe domain pattern the total energy of a bubble domain can also be written as a sum of domain wall energy, Zeeman energy, and magnetostatic energy.

The total energy of domain wall for an isolated bubble domain:

\[ e_w = 2\pi R t \gamma_w, \quad (1.65) \]

where \( R \) is the radius of bubble domain.

The contribution from the interaction between magnetic moments within the bubble domain volume and external field can be written as

\[ e_H = H M_s \pi R^2 t. \quad (1.66) \]

With respect to the scalar potential and the field outside the layer the bubble can be taken as a homogeneously magnetized cylinder of magnetization \( 2M_s \) (see Fig. 1.22). The function of charge distribution \( \sigma_b \) on the upper and lower planes of the bubble domain in cylindrical coordinate system is

\[
\sigma_b(r, z) = \begin{cases} 
\pm 2M_s & \text{for } z = 0, \\
-2M_s & \text{for } z = -t, \\
+2M_s & \text{for } 0 \leq r \leq R, \\
0 & \text{otherwise}.
\end{cases}
\]

Contrary to the infinite periodical stripe domain pattern an expression for the magnetostatic potential for an isolated bubble domain can be derived by straight integration in Eq. (1.14) over the surface of bubble domain with radius \( R \) [102]. Then the potential of the bubble stray field for \( z > 0 \) (see Fig. 1.22):

\[
\phi_b(r, z) = 2\pi \sigma R \int_0^\infty e^{-kz} J_1(kR) J_0(kr) \frac{dk}{k} \quad (1.68)
\]

Figure 1.22: (Color) a) The isolated bubble domain. The origin of the coordinate system \((z = 0, r = 0)\) is taken at the upper surface of the layer in the centre of the bubble. b) The hexagonal two dimensional lattice of bubble domains.
where

$$J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\tau - x \sin \tau) d\tau$$

is the Bessel function of \(n\)-order. Physical meaning of function \(\phi_b\) is the scalar potential of a flat circular disk of radius \(R\) and the constant charge density \(\sigma\) at distance \(z\) above the plane of the disk. Then, the total magnetostatic potential of a bubble domain, by analogy with the stripe domain pattern Eq. (1.43), can be represented as:

$$\Phi_b(r,z) = \begin{cases} 
\Phi^+_{b}(r,z) = -\phi_b(r, z) + \phi_b(r, z + t), & z > 0 \\
\Phi^d_{b}(r,z) = -\phi_b(r, -z) + \phi_b(r, z + t), & -t < z < 0 \\
\Phi^-_{b}(r,z) = -\phi_b(r, -z) + \phi_b(r, -z - t), & z < -t
\end{cases}.$$  (1.69)

It is easy to show that Eq.(1.69) fully satisfy Laplace equation (1.39) for the isolated bubble domain.

Using Eqs. (1.68,1.67) and (1.16) the magnetostatic self energy density of an isolated bubble domain in single magnetic layer can be expressed as

$$e_{m, b} = \int_0^R \int_0^{2\pi} (\sigma_b(r)\phi_b(r, 0) - \sigma_b(r)\phi_b(r, -t)) \, d\theta dx - 8\pi^2 M_s^2 R^2 t.$$  (1.70)

The second term in Eq. (1.70) corresponds to the interaction of the isolated bubble with the stray field of the homogeneously magnetized layer. After some transformation the magnetostatic energy of isolated bubble domain can be reduced to the following view (see Ref. [2], p. 313):

$$e_{m, b} = 4\pi^2 M_s^2 t^3 \left[ 1 + \frac{1 + d^2}{d^3} \left( [1 - d^2] E(u) - K(u) \right) \right]$$  (1.71)

where \(d = 2R/t\) is the reduced bubble diameter, \(u = d/\sqrt{1 + d^2}\) and \(E(u)\) and \(K(u)\) are the complete elliptical integrals defined as follows:

$$E(u) = \int_0^{\pi/2} \sqrt{1 - u^2 \sin^2 \alpha} d\alpha, \quad K(u) = \int_0^{\pi/2} d\alpha/\sqrt{1 - u^2 \sin^2 \alpha}.$$

Then the total energy of isolated bubble domain is

$$e_{tot, b} = 4\pi^2 M_s^2 t^3 \left( \frac{l_c}{l} \frac{H d^2}{8\pi M_s} + \frac{2d^3}{3\pi} \left[ 1 + \frac{1 + d^2}{d^3} \left( [1 - d^2] E(u) - K(u) \right) \right] \right).$$  (1.72)

Minimization of equation (1.72) with respect to \(d\) yields the equilibrium bubble diameter.

$$l_c t + \frac{H t d}{4\pi M_s} = F(d),$$  (1.73)

where \(F(d) = \sqrt{1 + d^2} E(u) - 2d/\pi\) is the so-called force function.

This implicit relation for \(d\) can be displayed graphically (Fig. 1.23). The values of \(l_c/h\) ratio and \(h = H/4\pi M_s\) define a straight line, which in general intersects the \(F(d)\) curve twice. The intersection with the larger \(d\) value yields the stable solution \(d_0\). Increasing the field causes
Figure 1.23: a) A diagram to derive bubble stability conditions. For a given $l_c/t$ and $h$ the (second) intersection of the straight line $l_c/t + hd$ with $F(d)$ yields the bubble diameter $d_0$. The intersection of the parallel to the $d$ axis through $l_c$ with the curves $S_{bc}(d)$ and $S_{bs}(d)$ yields the reduced bubble diameters $d_{bc}$ and $d_{bs}$, at bubble collapse and strip-out. The collapse field follows from the slope of the (tangential) line connecting $(0, l_c/t)$ and $(d_{bc}, F(d_{bc}))$. b) Energy profiles for isolated bubble domain as function of reduced diameter.

the bubble diameter to decrease until at a critical field the straight line just touches the $F(d)$ curve. This is the bubble collapse field $h_{bc}$. Bubble collapse can be described by the condition:

$$\frac{l_c}{t} = S_{bc}(d) = F(d) - d\frac{\partial F}{\partial d} = \frac{2d}{\pi} \left[d - \sqrt{1 + d^2}E(u) + \frac{u}{d^3}K(u)\right].$$  \hspace{1cm} (1.74)

The critical field $h_{bc}$ follows from (1.74) or graphically from the corresponding slope in the diagram (see Fig. 1.23). Finally, the stability against bubble strip-out is calculated by studying the bubble energy relative to an elliptic deformation [101]. It leads to the condition:

$$\frac{\pi l_c}{t} = S_{bs}(d) = \frac{1}{3} \left(S_{bc} - \frac{8d^2}{3\pi} \left[1 - \frac{1 + d^2}{d^3} \left\{(2 + d^2) E(u) - 2K(u)\right\}\right]\right).$$  \hspace{1cm} (1.75)

The upper, collapse field was first calculated by Kooy and Enz [22]. Thiele [101] added a derivation of the lower, strip-out field at which a bubble spontaneously expands into a band domain.

1.7.3 Magnetic Phase Diagram for Ferromagnetic Single Layer

It is useful to plot the critical fields for stripe and bubble patterns in one phase diagram. In Fig.1.24a we show the several characteristic fields for both stripe and bubble domain patterns depending on reduced thickness of magnetic layer.

The ground state of the magnetic layer with perpendicular anisotropy is the stripe domain pattern. Even for infinitely thin magnetic layer the stripe domains remain energetically favourable as compared with the bubble domains and the state with homogeneous magnetization.
Figure 1.24: (Color) Characteristic fields for domain patterns in a uniaxial plate as a function of the reduced length $t/(\pi l_c)$. The saturation field for the bubble lattice $h_{b0}$ is larger than $h_{s0}$, the saturation field for stripe domains. Individual bubbles are stable between $h_{bc}$ the bubble collapse field and $h_{bs}$, the strip-out field. $h_{tr}$ marks the boundary above which an optimized bubble lattice is energetically favoured relative to band domains [2].

When the reduced thickness $t/l_c$ decreases below the value of one, the saturation field for stripe domains (as well as for bubble) becomes small and the equilibrium domain width becomes large. This means that when magnetic layer become thinner than the characteristic length $l_c = \gamma/2K_d$, equilibrium domain patterns can hardly be formed in such films. The energy gain in the demagnetized state becomes very low for $t < l_c$ and any trace of wall coercivity will suppress the formation of equilibrium, regular domains. So there is a practical limit for domains in thin magnetic layers under the condition $t < l_c$. However, this empirical law does not apply for thin magnetic multilayers with strong perpendicular anisotropy, where the stripe domain period can exceed the thickness of the multilayer and regular domain pattern can exist even for $t < l_c$ [5]. Theoretically, also, there is no critical thickness which is limiting area for existence of regular stripe domains in perpendicular films [21, 22].

Let us briefly describe main features of the phase diagram in Fig. 1.24a. The characteristic field $h_{b0}$ is the field at which a bubble begins to carry a higher energy than the saturated film without a bubble (see $h_{b0}$ in Fig. 1.23b). The field $h_{b0}$ agrees with the stability limit (the saturation field) of the bubble lattice. Individual (noninteracting) bubbles are stable in a larger field range beyond and below the saturation field of the equilibrium bubble array beyond $h_{bs}$ up to $h_{bc}$. Strictly speaking, a bubble is also in a metastable state below $h_{b0}$ because a bubble lattice carries a lower energy in this field range. Practically bubbles can be treated as independent stable units in the field range between $h_{bs}$ and $h_{bc}$. This fact was utilized in magnetic bubble memory technology.

In Ref. [95] it was shown that at some critical field $h_{tr}$ the bubble lattice is energetically advantageous compared to the regular stripe pattern. But the energy difference is small, and
negligible as a driving force for domain rearrangement processes. The bubble lattice remains metastable at zero field and even at negative fields where it is converted into a network or “froth” structure that finally decays [103]. The ideal magnetization curve of a perpendicular film is shown in Fig. 1.24b. As there is no continuous path for a transition between the two domain patterns, no spontaneous conversion between band and bubble domain patterns is observed experimentally, and real magnetization curves follow mostly the continuous lines for either the stripe domain pattern or the bubble lattice pattern [2].
This chapter is concerned with exchange coupled multilayer. In the first section we describe effects of dipolar coupling between ferromagnetic layers in the frame of stripe and bubble domains models. Then we give comparison of our theoretical model with experimental results on [Co/Pt]/Ru multilayer. In the second section of this chapter we introduce and discuss our model of a lateral shift of domain walls in antiferromagnetically coupled multilayer. For this model we describe the impact of IEC on the ground state in a multilayer at zero external magnetic field. Finally the features of magnetization reversal processes in high-anisotropy perpendicular multilayer with IEC are given together with the comparison with experimental data.

2.1 Stripe and Bubble Domains in a Exchange Decoupled Ferromagnetic Multilayer

Usually nanostructured multilayers with perpendicular anisotropy consist of stacks of ferromagnetic bilayers $F(t)/M(s)$ or “ferromagnetic” blocks $[F(t)/M(s)]_X$ antiferromagnetically coupled via $A(S)$ spacers ($F=\text{Co, Fe, M=Pt, Pd, A= Ru, NiO, Ir}$) with constant or variable thicknesses of ferromagnetic $t$ and nonmagnetic $s$, $S$ layers [7,19,20,104–107]. These systems can be described within a general model composed of $X \times N$ identical magnetic layers of thickness $t$ separated by $X \times (N-1)$ nonmagnetic spacers of thickness $s_i$. Particularly, this includes superlattices $\{(Co(t)/M(s))_{X-1}/Co(t)/A(S)\}_N$ including $X \times N$ Co layers combined into $N$ ferroblocks antiferromagnetically coupled via $N-1$ A-spacers (see e.g. Refs. [7, 19, 20, 104–107]). First let us consider the simplest case of exchange decoupled multilayer where only long-range dipolar interaction between the layers is taken into account.

2.1.1 Stripe Domains in Multilayers

The stripe domain model in a multilayer is based on the same assumptions as a one-dimensional stripe domain model for an isolated magnetic layer (see Sect. 1.7.1). We assume that in each ferromagnetic layers the magnetization $M$ varies along one of the axes ($x$-axis here). This multidomain structure consists of homogeneously magnetized stripes with up and down magnetization $M \equiv |M_x| = \text{const}$ (Fig. 2.1). Magnetic inhomogeneities are localized in transition regions, i.e., domain walls between adjacent domains. Likewise in the Sect. 1.7.1, it is assumed that these walls are negligibly thin compared to the stripe widths. However, the domain walls contribute a positive excess or defect energy with a surface energy density $\gamma_w$ (see Sects. 1.6). In order to minimize the magnetostatic energy interaction between all magnetic layers it is assumed that the domain walls throughout the whole stack of the ferromagnetic layers sit
Figure 2.1: (Color) a) Two ferromagnetic layers of thickness \( t \) separated by distance \( \omega t = a + t \) (a - interlayer thickness) and with parallel arrangement of the magnetization introduce geometrical parameters of stripes. b) This sketch illustrates dipolar coupling between two magnetic layer which appears due to surface charge interaction.

exactly on top of each other, and each planar domain wall belongs to the plane parallel to \( yz \) (see Fig. 2.1). We will show later (see Sect. 2.2) that this assumption is valid only for decoupled (or ferromagnetically coupled) multilayer and strictly speaking is not correct for antiferromagnetically coupled multilayer. Here we use the same notations as in in Sect. 1.7.

The magnetostatic energy of a multilayer can be decomposed into magnetostatic self energy \( e_s \) of each magnetic layer and interaction energy \( e_i \) between all of them. The magnetostatic self energy of each magnetic layer is defined by Eq. (1.46). To define the interaction energy first let us consider two magnetic layers at the distance \( \omega t \) between their centres scaled by the thickness of layers. In other words \( \omega = (t + a)/t \), where \( a \) is the thickness of interlayer between two magnetic layers (see Fig. 2.1). The stray field produced by magnetic charges on the surfaces of one layer penetrate into the volume of another one. The energy of magnetic charges of one layer in the potential of another layer is the magnetostatic interaction energy. Then according to the Eq. (1.16) the interaction energy between two magnetic layer at the distance \( \omega t \) can be written as

\[
e_{i,2} = \frac{1}{2} \int_0^\pi [\sigma_s(x) \phi_s(x, \omega t) + \sigma_s(x) \phi_s(x, \omega t - t) - \sigma_s(x) \phi_s(x, \omega t - t) - \sigma_s(x) \phi_s(x, \omega t + t)] dx, \tag{2.1}
\]

where \( \sigma_s(x) \) and \( \phi_s(x, z) \) are defined by Eqs. (1.40) and (1.44) respectively. The first, second, third and fourth terms in Eq. (2.1) are the interaction energies between 1–3, 2–4, 2–3, and 1–4 charged surfaces respectively (see Fig. 2.1b). After integration the interaction energy \( e_i \) between two magnetic layer at the distance \( \omega t \) can be written as

\[
e_{i,2} = 2\pi M_s^2 t \left( \frac{4}{\pi^2 p} \sum_{n=1}^\infty \frac{1}{n^3} \left(1 - (-1)^n \cos (\pi nm)\right) \left[2e^{-n\omega p} - e^{-(n-1)\omega p} - e^{-n(\omega+1)p}\right]\right), \tag{2.2}
\]

where \( m = (d_+ - d_-)/D \) – reduced magnetization, \( p = 2\pi t/D \) – parameter inversely proportional to the reduced period. Using equation (1.50) the infinite sums in Eq. (2.2) can be reduced into finite integrals. Indeed, after simple algebraic manipulation Eq. (2.2) can be
written as follows

$$e_{i,2} = -2\pi M_s^2 t \left( 2\Omega_s(\omega) - \Omega_s(\omega+1) - \Omega_s(\omega-1) \right),$$  \hspace{1cm} (2.3)

with

$$\Omega_s(\omega) = \omega \left( m^2 + \frac{4}{\pi^2 \omega p} \sum_{n=1}^{\infty} \frac{1}{n^3} \left\{ 1 - (-1)^n \cos(\pi nm) \right\} [1 - \exp(-n\omega p)] \right).$$  \hspace{1cm} (2.4)

Finally according to identity Eq. (1.50) one can write integral representation for this function

$$\Omega_s(\omega) = \omega - \frac{2\omega^2 p}{\pi^2} \int_0^1 (1 - \xi) \ln \left[ 1 + \frac{\cos^2(\pi m/2)}{\sinh^2(\omega p\xi/2)} \right] d\xi.$$  \hspace{1cm} (2.5)

Strictly speaking function $\Omega_s = \Omega_s(p, m, \omega)$ depends on three variable. But for simplicity we use the short notation $\Omega_s = \Omega_s(\omega)$ unless we want to refer to the explicit dependence on $p$ and $m$.

The negative or positive sign of the energy contribution in Eq. (2.3) depends on the sign of charge of corresponding interacting planes. If the interacting planes $i$ and $j$ have equal signs $\sigma_{s,i}(x) = \sigma_{s,j}(x)$ (as planes 1–3 and 2–4, see Fig. 2.1b) the energy contribution has negative sign. If not $\sigma_{s,i}(x) = -\sigma_{s,j}(x)$ (as planes 2–3 and 1–4) the interaction energy contribution has positive sign. Self energy of each magnetic layer resulting from interaction between planes as 1–2 and 3–4 is just $\Omega(1)$.

The interaction energy in a multilayer, then, can be written as sum of interactions between all pairs of magnetic layers. The most general case of multilayer of $X$ ferromagnetic layers with equal thickness $t$ separated by interlayers with different thickness $s_i$ (see Fig. 2.2a) can be written as:

$$e_i = -\sum_{k=1}^{X-1} \sum_{j=k+1}^{X} \left[ 2\Omega_s(\omega_{kj}) - \Omega_s(\omega_{kj}+1) - \Omega_s(\omega_{kj}-1) \right],$$  \hspace{1cm} (2.6)
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where $\omega_{kj} = \sum_{i=k}^{j-1} [1 + s_i/t]$ and $s_i$ is the thickness of $i$-th interlayer (see Fig. 2.2a). For the case of equal interlayer thicknesses the interaction energy Eq. (2.6) can be simplified:

$$e_i = -\sum_{k=1}^{X-1} (X - k) [2\Omega_s(\omega k) - \Omega_s(\omega k + 1) - \Omega_s(\omega k - 1)].$$  \hspace{1cm} (2.7)

Factor $(X - k)$ in Eq. (2.7) appear due to the equivalence of interaction energy for magnetic layers at equal distances (see Eq. (2.3) and Fig. 2.2b). For a multilayer with fixed thicknesses of magnetic later $t$ and interlayer $s$ we use the terms stack or ferro block. The total energy of such a ferromagnetic stack of $X$ magnetic layers reduced to $2\pi M_s^2 t X$ is

$$e_{tot} = \Omega_s(1) - \frac{1}{X} \sum_{k=1}^{X-1} (X - k) \Xi_{\omega k} + \frac{2l_c p}{\pi t} - \frac{H m}{2\pi M_s}.$$ \hspace{1cm} (2.8)

Here and below we use the following notation

$$\Xi_F(x) = 2F(x) - F(x+1) - F(x-1).$$  \hspace{1cm} (2.9)

Minimization of the total energy in Eq. (2.8) with respect to $p$ and $m$ leads to a system of equations that determine the equilibrium parameters of stripe domains in the stack. This system of equations can be written as

$$\frac{\partial \Omega_s(1)}{\partial p} - \frac{1}{X} \sum_{k=1}^{X-1} (X - k) \Xi_{\omega k} = -\frac{2l_c}{\pi t},$$ \hspace{1cm} (2.10)

$$\frac{\partial \Omega_s(1)}{\partial m} - \frac{1}{X} \sum_{k=1}^{X-1} (X - k) \Xi_{\omega k} = \frac{H}{2M_s},$$ \hspace{1cm} (2.11)

with

$$\frac{\partial \Omega_s(\omega)}{\partial p} = -\frac{2\omega^2}{\pi^2} \int_0^1 \xi \ln \left[ 1 + \frac{\cos^2(\pi m/2)}{\sinh^2(\omega \xi p/2)} \right] d\xi$$

and

$$\frac{\partial \Omega_s(\omega)}{\partial m} = \frac{\omega^2 p \sin(\pi m)}{\pi} \int_0^1 \frac{(1 - \xi) d\xi}{\sinh^2(\omega p \xi/2) + \cos^2(\pi m/2)}.$$

In Ref. [1] we have shown that the dependence of stripe domain period on magnetic layer thickness in multilayer exhibit unusual features. In particular, Fig. 2.3 shows the typical dependences of the period $D$ of stripe domains in the ferromagnetic stack on the magnetic layer thickness $t$, which were calculated using Eqs. (2.10) for fixed values of ratio $\nu = s/t$ in the absence of an external magnetic field ($m \equiv 0$). The solutions for ferromagnetic domains exist for any thickness of the magnetic layer. The domain period increases with the magnetic layer thickness. This behavior is retained down to the $t \rightarrow 0$. For the different $\nu$ value, the balance between magnetostatic energy and domain wall energy changes. In
Figure 2.3: (Color) Dependences of the stripe domain period $D$ on the magnetic layer thickness for multilayer with $X = 2$ (a) and $X = 10$ (b) and various values of $\nu = s/t$.

both limiting cases of relatively small ($s \ll t$) and large ($s \gg t$) thicknesses of nonmagnetic spacers, the dependence $D(t)$ reflects the well-known behavior of a isolated layer (see Sect. 1.7.1, Fig. 1.20). In the limit of small $\nu$, the period $D(t)$ of stripe domains approaches the value for a single magnetic layer with an effective total thickness $Xt$. For very large distances between magnetic layers ($\nu \gg 1$), the period of stripe domains is determined by the properties of isolated (noninteracting) layers of thickness $t$. However, for the finite (intermediate) $\nu$ values, a strong magnetostatic interaction between the neighboring magnetic layers significantly complicates the $D(t)$ function compared to the $D(t)$ lines for separate layers (Fig. 2.3). The $D(t)$ functions for a superlattice are strongly nonmonotonic and exhibit extrema and inflection points. According to Eq. (2.10) the implicit function for dependence $D(t)$ can be written as

$$f_0(t,D) = \frac{\partial \Omega_{\nu}(1)}{\partial p} - \frac{1}{X} \sum_{k=1}^{X-1} (X-k) \Xi_{\omega m}(\omega k) + \frac{2l_c}{\pi t} = 0. \quad (2.12)$$

From the properties of an implicit function we get the condition for its extrema

$$\frac{dD}{dt} = \frac{\partial f_0(t,D)}{\partial t} = 0. \quad (2.13)$$

The latter is valid if $\partial f_0(t,D)/\partial t = 0$. Then, after some algebra the equation for extremuma of the function $D(t)$, at $H = 0$, can be written as

$$\Upsilon(1) - \frac{1}{X} \sum_{k=1}^{X-1} (X-k) \Xi_{\nu}(\omega k) = 0 \quad (2.13)$$

with

$$\Upsilon(\omega) = 2\omega^2 \ln \left[ \tanh \left( \frac{\omega p}{2} \right) \right] - \frac{\partial \Omega(\omega)}{\partial p} \bigg|_{m=0}. \quad (2.13)$$
Figure 2.4: (Color) Dependences of the stripe domain period $D$ on the magnetic layer thickness for multilayer with $X = 5$ for different values of $\nu = s/t$ (solid black lines). The open circles correspond to two inflection points with coordinates $(t/l_c, D/l_c) = (1.19, 37.9)$ for the “first” and $(21.8, 43.3)$ for the “second” inflection point which satisfy Eq. (2.14). The dotted lines are the trace of two minima (blue line for “first” minimum and green line for “second” minimum) and one maximum (red line). Each solid line corresponding to the case of: (i) one (first) minimum only ($\nu = 0.001$), (ii) one (first) minimum and one (second) inflection point ($\nu = 0.282$), (iii) intermediate case of two minima and one maximum ($\nu = 0.95$), (iv) one (second) minimum and one (first) inflection point ($\nu = 2.275$), and (v) one (second) minimum only ($\nu = 5$).

The equation for inflection points at the dependence $D(t)$ have the following form

$$\Upsilon^*(1) - \frac{1}{X} \sum_{k=1}^{X-1} (X - k) \Xi_{\Upsilon^*}(\omega k) = 0 \quad (2.14)$$

with

$$\Upsilon^* (\omega) = 2\omega^3 / \sinh (\omega p/2) - p^{-1} \frac{\partial \Omega(\omega)}{\partial p} \bigg|_{m=0}.\)$$

In Fig. 2.4 we show the dependence of stripe domain period on the thickness of magnetic layer for multilayer with $X = 5$ and different values of $\nu$ together with the two inflection points and trace of extremum points for varying values of $\nu \in [0, \infty]$.

In recent years, much attention has been devoted to thin films and superlattices based on the Co/Pt system [5, 7, 20, 105]. In particular, Hellwig et al. [7] studied the magnetic properties and domain structures of $[\text{Co}(4\, \text{Å})/\text{Pt}(7\, \text{Å})]_X$ multilayers. Figure 2.5 shows the experimental plot of the domain size $D/2$ versus the number of magnetic layers in such systems in comparison to the results of calculations using Eq. (2.10). The calculations were performed using the mean square value of the characteristic length $l_c$ for experimental points (see the inset in Fig. 2.5).

In the Sect. 1.7.1 we have introduced approximate models. In particular Kittel model for
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Figure 2.5: Dependence of the domain size $D/2$ on the number of magnetic layers $X$ in [Co(4A)/Pt(7A)]$_X$ multilayers: (open circles) experimental data [7]; (black squares) calculations were performed using formulas Eq. (2.10) with a mean square value of the characteristic length $l_c = 4.432$ nm for experimental points (inset).

thick magnetic layers and models for the approximation of large domains, so-called Model I and II. Here we extend approach of Model I and II to the case of a multilayer. Investigation of such a limiting case when period of domains is much larger than thickness of magnetic layer is important for further discussion of the ground states and magnetization reversal behaviour in magnetic multilayer.

**Model I.** For a thin magnetic layer when the period of domains exceeds the thickness the expression for magnetostatic energy of the stripe domain pattern can be considerably simplified. As well as for isolated magnetic layer (see Sect. 1.7.1) for the case of $D \geq t$ the function $\sinh(\omega p \xi/2)$ in Eqs. (2.5), can be replaced by its arguments. After that all integrals in Eq. (2.8) can be evaluated in terms of elementary functions and the interaction energy of two charged planes $\Omega_s(\omega)$ can be reduced into the form

$$\Omega_{s,1}(\omega) = \omega + \frac{2\omega^2 \cos(\pi m/2)}{\pi^2 b} \cdot V(b/\omega) \tag{2.15}$$

where $b = 2 \cos(\pi m/2)/p$ and $V(x) = (x^2 - 1) \ln(x^2 + 1) - x^2 \ln x^2 - 4x \arctan(1/x)$. Then the total energy of the multilayer become

$$e_{tot,1} = 1 + \frac{2 \cos(\pi m/2)}{\pi^2 b} \left( \tilde{V}(b) - \frac{1}{X} \sum_{k=1}^{X-1} (X - k) \Xi\tilde{V}(\omega k) + \frac{2\pi l_c}{t} \right) - \frac{Hm}{2\pi M_s}, \tag{2.16}$$

where $\tilde{V}(b/\omega) = \omega^2 V(b/\omega)$. 

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The system of equations for the equilibrium parameters $b$ and $m$ takes the following form:

$$\frac{\partial \Omega_{s,I}(1)}{\partial b} - \frac{1}{X} \sum_{k=1}^{X-1} (X - k) \Xi_{\sigma_{\alpha_{ij}}}(\omega k) = \frac{2l_c}{\pi t}$$  \hspace{1cm} (2.17)

$$\frac{\partial \Omega_{s,I}(1)}{\partial m} - \frac{1}{X} \sum_{k=1}^{X-1} (X - k) \Xi_{\sigma_{\alpha_{ij}}}(\omega k) = \frac{H}{2M_s}$$  \hspace{1cm} (2.18)

with

$$\frac{\partial \Omega_{s,I}(1)}{\partial b} = \frac{\omega^2}{2} \left( \ln \left[ 1 + \left( \frac{b}{\omega} \right)^2 \right] + \left( \frac{b}{\omega} \right)^2 \ln \left[ 1 + \left( \frac{\omega}{b} \right)^2 \right] \right)$$

and

$$\frac{\partial \Omega_{s,I}(1)}{\partial m} = \omega^2 \sin \left( \frac{\pi q}{2} \right) \left( 2 \arctg \left( \frac{\omega}{b} \right) - \left( \frac{b}{\omega} \right) \ln \left[ 1 + \left( \frac{\omega}{b} \right)^2 \right] \right).$$

The critical field for the transition to a homogeneous state $H_{s0}$ can be obtained from the system of equations (2.17) and (2.18) for $m = 1$

$$H_{s0,X} = 4M_s \left[ \Psi(b, 1) - \frac{1}{X} \sum_{k=1}^{X-1} (X - k) \Xi_{\psi}(\omega k) \right]$$  \hspace{1cm} (2.19)

where $\Psi(b, \omega) = \omega \left( 2 \arctg \left( \frac{\omega}{b} \right) - \left( \frac{b}{\omega} \right) \ln \left[ 1 + \left( \frac{\omega}{b} \right)^2 \right] \right)$, and $b$ obeys the equation (2.17). The transition field $H_{s0,X}$ (Eq. (2.19)) and $H_{s0}$ (Eq. (1.59)) converge for the case $X = 1$. The analytical solutions for the equilibrium magnetization $m(H)$ and domain size $D(H)$ and $d_\pm(H)$ according to Eqs. (1.60) and (1.61) can be written as

$$m = \left( \frac{2}{\pi} \right) \arcsin \left( \frac{H}{H_{s0,X}} \right),$$  \hspace{1cm} (2.20)

$$D(H) = \pi tb / \sqrt{1 - \left( \frac{H}{H_{s0,X}} \right)^2}, \quad d_\pm = \left( \frac{D}{\pi} \right) \arccos \left( \mp \frac{H}{H_{s0,X}} \right),$$  \hspace{1cm} (2.21)

where the relation between the characteristic field $H_{s0,X}$ and the material parameter $l_c/t$ is provided by the system of equations (2.19) and (2.17). The equations for the magnetic susceptibility of ferromagnetic multilayer can be written in the same form as Eq. (1.63) where $H_{s0}$ should be replaced by $H_{s0,X}$.

Figure 2.6 shows a comparison of the transition field $H_{s0,X}$ calculated as a function of the parameter $t/l_c$ using Eq. (2.19) for three model systems.

In the entire range of variation of the $t/l_c$ value, the $H_{s0,X}$ curve for a bilayer with $\nu = 0.5$ occurs in the middle between the curves for a single layer of thickness $t$ and a single layer of double thickness $2t$. As the magnetic layer thickness $t$ decreases, the $H_{s0,X}$ value for the bilayer approaches the value for the monolayer of double thickness. As the layer thickness $t$ grows, the transition field for the bilayer tends to the $H_{s0,X}$ value for the single layer. This behavior is explained by the fact that, for a fixed ratio $\nu = s/t$, a decrease (increase) in the magnetic layer thickness $t$ implies the corresponding decrease (increase) in the nonmagnetic layer thickness $s$ which, in turn, results in an increase (decrease) of the contribution for the interlayer interaction energy as compared with the magnetostatic self energy of each layer. These features in the behavior of the critical field $H_{s0,X}$ must also be observed in multilayer
Figure 2.6: Plots of the field \( h_{s0,X} = H_{s0,X} / (4\pi M_s) \) corresponding to a transition of the stripe domain structure to a homogeneous state for (1) a ferromagnetic single layer, (2) a single layer of double thickness \((\nu \to 0)\), and (3) a bilayer with \( \nu = s/t = 0.5 \).

ferromagnetic systems.

**Model II.** For the thin layer when \( t/D \ll 1 \) \((p \ll 0)\) further simplifications of the stripe solutions can be performed. After expansion for large \( b \) of the function \( \tilde{V}(b/\omega) \) in Eq. (2.16) it can be expressed as

\[
\tilde{V}(b/\omega) = -\omega^2 \left[ 3 + \ln \left( \frac{b^2}{\omega^2} \right) \right].
\]

(2.22)

Then the total energy of the system takes the form

\[
e_{\text{tot, II}} = 1 - \frac{4X \cos(\pi m/2)}{\pi^2 b} \left( \frac{3}{2} + \ln \left( \frac{b/X}{\Lambda} \right) \right) - \frac{Hm}{2\pi M_s},
\]

(2.23)

where \( \Lambda = \pi l_c/(Xt) - \ln(X) + X^{-2} \sum_{k=1}^{X-1} (X - k) \Xi_v(\omega k), \ \nu(\omega) = \omega^2 \ln(\omega). \) Minimization of energy (2.23) yields the analytical solution for the transition field and parameter \( b \):

\[
H_{s0,X}^* = 4M_s \exp(1/2 - \Lambda), \quad b = X \exp(\Lambda - 1/2)
\]

(2.24)

and then using Eq. (2.21) one can get the analytical solutions for \( D(H) \):

\[
D(H) = \frac{\pi t X \exp(\Lambda - 1/2)}{\sqrt{1 - (H/H_{s0,X}^*)^2}}.
\]

(2.25)

Another widely used approach is the so-called model of single layer (see e.g. Refs. [90] and [20]). In this model the multilayer of \( X \) magnetic layers thickness \( t \) and \( X - 1 \) interlayers thickness \( s \) is described as one single layer of total thickness \( T = Xt + (X-1)s \). The saturation magnetization \( \tilde{M}_s = M_s \cdot X / [X + (X-1)\nu] \) and domain wall energy \( \tilde{\gamma}_w = \gamma_w \cdot X / [X + (X-1)\nu] \) are assumed to be averaged over the whole of the multilayer. In such an approach, the solution for the domain size and transition fields are defined with the same equations which was introduced for isolated layer in Sects. 1.7.1 and 1.7.2 where the reduced parameters \( T, L_c = \tilde{\gamma}_w/(4\pi \tilde{M}_s^2) = l_c \cdot [X + (X-1)\nu] / X, \) and \( P = 2\pi T / D = p \cdot [X + (X-1)\nu] \) should be used.
The relative error $\epsilon_{\Delta e} = (\Delta e/e_{\text{tot}}) \times 100\%$ as function of reduced thickness $t/l_c$ for the multilayer with $X = 10$ and different ratio $\nu = s/t$. The calculation has been done for the case of zero external field ($H = 0, m \equiv 0$).

But for the comparison of this approach with the rigorous solution it is more convenient to deal with the same variable in both cases. The total energy of the system can be written as

$$e_{\text{tot}, \text{sl}} = 1 - \frac{2\hat{\omega}p}{\pi^2} \int_0^1 (1 - \xi) \ln \left[1 + \cos^2 \left(\frac{\pi m/2}{\sinh^{\frac{\xi}{2}}[\hat{\omega}p/2]}\right)\right] d\xi + \frac{2l_c\hat{\omega}p}{\pi tX} - \frac{Hm\hat{\omega}}{2\pi M_sX} \Omega_s(\hat{\omega}p, m, 1) + \frac{2l_c\hat{\omega}p}{\pi tX} - \frac{Hm\hat{\omega}}{2\pi M_sX},$$

where $\hat{\omega} = \omega(X - 1) + 1$. Below we will point out under what conditions this model fit the rigorous solution.

The total energy of the multilayer is defined with Eq.(2.8) where the magnetostatic energy is composed of the sum of interaction energy $\Omega$ between $2X$ charged planes which generally is a function $\Omega(p, m, \omega)$ of three parameters $p, m$ and $\omega$. The total energy Eq.(2.8) after some algebra can be be decomposed into two terms as follows

$$e_{\text{tot}} = e_{\text{tot}, \text{sl}} + \Delta e.$$

Here $e_{\text{tot}, \text{sl}}$ is the total energy of a single layer model defined by Eq. (2.26) and

$$\Delta e = \left[\Omega(p, m, 1) - \frac{1}{X} \sum_{k=1}^{X-1} (X - k) \Xi_{\Omega}(\hat{\omega}k) - \Omega(p[\omega(X - 1) + 1], m, 1)\right] - \left[\frac{2l_c\hat{\omega}p}{\pi tX} - \frac{Hm\hat{\omega}}{2\pi M_sX}\right] \frac{X - 1}{X} \nu.$$

It is easy to show that function $\Delta e \to 0$ for $\nu \to 0$. This limits holds because the internal charges compensate each other when the interlayer thickness goes to zero. Figure 2.7 shows the dependences of relative error $\epsilon_{\Delta e} = (\Delta e/e_{\text{tot}})$ as function of the reduced thickness $t/l_c$ for different value of parameter $\nu = s/t$ and zero external field. For each value of the thickness in Fig. 2.7 we define the equilibrium parameter $p$ from the rigorous solution Eq. (2.10). As seen
from the figure, for the small value of reduced thickness of magnetic layer \( t/l_c < 0.1 \) the contribution from \( \Delta e \) becomes insignificant \( (\epsilon_{\Delta e} \ll 1\%) \) even for \( \nu > 1 \). At the same time for small \( \nu \) the thickness interval where relative error becomes small is much wider. For example at \( \nu = 1/10 \) the relative error is less than 1\% for all values of \( t/l_c < 0.4 \). Thus we can neglect \( \Delta e \) and use the single layer model in both limiting cases for \( s/t \rightarrow 0 \) and for \( t/l_c \ll 1 \).

Now let us consider how the approaches of Model I, II and single layer model fit the rigorous solution. In Fig. 2.8 we show the rigorous solution for the stripe domain period at zero magnetic field for multilayer with \( X = 10 \) and two values of parameter \( \nu = s/t \) together with the solutions for Models I, II and single layer model. As well as for the isolated magnetic layer the solution of Models I and II converge toward the rigorous solution with decreasing magnetic layer thickness. For the chosen value \( \nu = s/t = 7/4 \) and 2/3 relative error for Models I and II with respect to rigorous solution is less than 1\% for the reduce thickness \( t/l_c \leq 0.1 \). Contrary to the Models I and II the single layer model does not converge exactly to the rigorous solution for \( t \rightarrow 0 \) (see the arrows in Fig. 2.8) but exhibit comparably small value of relative error of about \( \pm 1\% \) for chosen value \( \nu \) and \( t/l_c \leq 0.1 \). The limiting condition for Model I and II is only the reduced thickness \( t/l_c \ll 1 \) while the useable area of the single layer model in addition depends on \( \nu \) and \( X \).

In the following chapters we use the single layer model approach to simplify the general model of antiferromagnetically coupled multilayer. Thus, the useable area of this model should be discussed in more detail. In Fig. 2.9 a we show the dependence of relative error of single layer model with respect to the rigorous solution as a function of material parameters \( \nu \) and \( t/l_c \) for fixed value \( X = 10 \). As was mentioned above the relative error has the lowest value for the small \( \nu \) and \( t/l_c \). In particular for \( X = 10 \) the relative error does not exceed \( \pm 1\% \) for a wide interval of \( 0 < \nu \leq 1 \) and \( 0 < t/l_c \leq 0.5 \). For the reduced thickness interval \( 0 < t/l_c \leq 0.2 \) the relative error does not exceed \( \pm 2\% \) even for wider range of \( \nu \) from 0 up to 2. Figure 2.9 a reflect general features of the single layer model but could be slightly changed with number of magnetic layers \( X \) which is also the parameter of the system. In Fig. 2.9 b the dependence of relative error is showed as function of reduced thickness \( t/l_c \).
Figure 2.9: (Color) Relative error maps for the solution for the stripe domain period at zero field in single layer approach compare to the rigorous solution of multilayer defined with Eq. (2.10). a) Relative error as a function of reduced thickness $t/l_c$ and $\nu = s/t$ for multilayer with $X=10$. b) Relative error as a function of reduced thickness $t/l_c$ and number of magnetic layers $X$ at fixed $\nu = s/t = 7/4$. The color-coded error value scales are shown right of pictures.

and number of magnetic layers $X$ for the fixed value of $\nu = 7/4$. This value of parameter $\nu$ corresponds to the case of $[\text{Co}(4\,\text{Å})/\text{Pt}(7\,\text{Å})]_X$ multilayer which we used as a main model for comparison of our theoretical results with experimental data. As it follows from Fig. 2.9b for $0 < t/l_c \leq 0.2$ the relative error for the single layer model does not exceed 3% and rapidly decrease with an increase in the number $X$. Finally, all above can summarized as follows. (i) Strictly speaking the single layer model converges to rigorous solution only for limiting case $\nu \to 0$. ii) However, the relative error of this model remains very small (about $\pm 2\%$) in wide range of parameters. In particular for $[\text{Co}(4\,\text{Å})/\text{Pt}(7\,\text{Å})]_X$ multilayer it is valid to apply this approach with following value of the system parameters: $0 < t/l_c \leq 0.2$, $0 \leq \nu \leq 2$, $X \geq 5$.

### 2.1.2 Bubble Domains in Multilayers

Now let us consider an isolated bubble of radius $R$ (Fig.2.10). We assume that, to minimize the magnetostatic energy, the bubble domains in the whole multilayer sit exactly on top of each other. Then the same reasonings as for the case of stripe domains described in detail in the previous subsection (see Eqs. (2.3) – (2.5) and related text) lead to the following. The interaction energy between two magnetic layers for the case of isolated bubble domains can be written as

$$e_{1,2} = -4\pi^2 M_s^2 t^3 \left(2\Omega_b(\omega) - \Omega_b(\omega + 1) - \Omega_b(\omega - 1)\right),$$

with

$$\Omega_b(\omega) = \frac{2}{\pi^3} d^3 \left[1 + \sqrt{1 + \left(\frac{d}{\omega}\right)^2} \left(1 - \left(\frac{d}{\omega}\right)^2\right) E\left(u\left(\frac{d}{\omega}\right)\right) - K\left(u\left(\frac{d}{\omega}\right)\right)\right]$$

(2.29)
where \( u(x) = x/\sqrt{1+x^2} \) and \( \omega = s/t + 1 \) is the reduced distance between the centres of magnetic layer separated with interlayer thickness \( s \). Equation (2.29) for \( \Omega_b(\omega) \) which can be considered as the interaction energy of two charged disks at the distance \( \omega t \) is obtained from magnetostatic energy of isolated bubble domain Eq. (1.71) by the replacement of \( t \) by \( \omega t \). In this case function \( \Omega_b(\omega) \) contains additional terms of self energy of charged discus and the energy of bubble domain of radius \( R \) and thickness \( t \) in the stray field of homogeneously magnetized layer of the same thickness \( \omega t \). In particular, for \( \omega = 1 \) function \( \Omega_b(1) \) exactly reduce to the magnetostatic self energy of an isolated bubble (see Eq. (1.71)). However as the first additional term does not depend on the distance at all (self energy of charged disks) and the second one is linearly dependent on the distance between interacting surface charges \((−8\pi^2M_s^2R^2\omega t, \text{see Eq. (1.70)}) \) after the summation in Eq. (2.28) both contributions vanish. Thereby, only \( \Omega_b(1) \) and the complete sum of the function in Eq. (2.28) have a rigorous physical meaning, in contrast to the functions \( \Omega_b(\omega) \).

By analogy with the stripe domains the magnetostatic energy of a multilayer in general case of equal thickness of magnetic layers \( t \) and different thickness of interlayers defined as \( s_i \) can be written as:

\[
e_{m,b} = \Omega_b(1) - \sum_{k=1}^{X-1} \sum_{j=k+1}^{X} \Xi_{\Omega_b}(\omega_{kj}),
\]

where \( \Xi_{\Omega_b}(x) = 2\Omega_b(x) - \Omega_b(x+1) - \Omega_b(x-1) \), \( \omega_{kj} = \sum_{i=k}^{j-1} [1 + s_i/t] \) and \( s_i \) is the thickness of \( i \)-th interlayer. For the case of equal interlayer thicknesses the interaction energy Eq. (2.30) can be simplified:

\[
e_{m,b} = \Omega_b(1) - \sum_{k=1}^{X-1} (X - k) \Xi_{\Omega_b}(\omega).
\]

The total energy of an isolated bubble domain in the multilayer is

\[
e_{\text{tot},b} = 4\pi^2M_s^2\mu^3 \left( \frac{d_{\text{eff}}}{t} + \frac{HD_s^2}{8\pi M_s} + e_{m,b} \right).
\]

The equilibrium bubble diameter is derived by minimization of energy \( e_{\text{tot},b} \) (2.32) with respect to \( d \). In particular, for multilayers with equal spacers the equilibrium \( d \) is derived
from the following equation

\[
\frac{l_c}{t} + \frac{H d}{4\pi M_s} = F(1) - \sum_{k=1}^{X-1} (X - k) \Xi F(\omega)
\]  (2.33)

where

\[
F(\omega) = -\frac{\partial \Omega_b(\omega)}{\partial d} = -\frac{2d^2}{\pi} \left(1 - \frac{E(u(d/\omega))}{u(d/\omega)}\right).
\]  (2.34)

In Sect. 1.7.2 we showed that the existence of isolated bubbles is restricted by strip-out instability at lower fields and a collapse at high fields. In Ref. [II] we have shown that a similar situation occurs for bubbles in multilayers. In particular, following the standard methods (see e.g. Ref. [2]) one can derive the critical parameters of the bubble existence based on two stability criteria. For multilayers by analogy with the case of single layer Eq. (1.74) the equation for the critical minimum diameter (collapse diameter) \(d_{bc}\) is defined as

\[
\frac{l_c}{t} = S_{bc}(1) - \sum_{k=1}^{X-1} (X - k) \Xi S_{bc}(\omega).
\]  (2.35)

For a film with given architecture \((t, s, X)\) and materials properties \((l_c)\) an isolated bubble is stable only with \(d > d_{bc}\). Furthermore, the analysis of elliptical distortion results in an upper critical diameter \(d_{bs}\), defined by following equation

\[
\frac{l_c}{t} = S_{bs}(1) - \sum_{k=1}^{X-1} (X - k) \Xi S_{bs}(\omega).
\]  (2.36)

The stability functions \(S_{bc}\) and \(S_{bs}\) for the case of multilayers are given as:

\[
S_{bc}(\omega) = \frac{2d}{\pi} \left[ d - \sqrt{1 + \frac{d^2}{\omega^2}} E(u(d/\omega)) + \frac{u(d/\omega)}{d} K(u(d/\omega)) \right], \quad (2.37)
\]

\[
S_{bs}(\omega) = \frac{2d}{9\pi} \left[ d - \frac{d}{u(d/\omega)} \left(1 + \frac{8\omega^2}{d^2}\right) E(u(d/\omega)) + \frac{u(d/\omega)}{d} \left(5 + \frac{8\omega^2}{d^2}\right) K(u(d/\omega)) \right]. \quad (2.38)
\]

Within the limits given by \(d_{bc}\) and \(d_{bs}\) a cylindrical bubble is stable and its size is an unambiguous function of the external field according to equation (2.33).

2.1.3 Comparison between theoretical model and experimental results

In this chapter, we describe the domain structure and the magnetization processes in the \([\text{Co/Pt} ]_{X-1}/\text{Co/Ru}\}_N\) multilayer with \(X = 8\) and \(N = 18\). The zero field state is characterized by ferromagnetic stripe domains also common for simple single layer films with perpendicular anisotropy. As shown in Ref. [III] the AF coupling via Ru interlayer violates the stripe coherency through the stack inducing a relative transverse shift of domains in the adjacent layers. However, for multilayers with the FM stripe ground state this shift is very small and can be neglected.
The multilayer system used for our experiments is \([\{\text{Co}(4\AA)/\text{Pt}(7\AA)\}]_{X-1}/\text{Co}(4\AA)/\text{Ru}(9\AA)\)\(^N\), which is in the following referred to as the [Co/Pt]/Ru multilayer. Details on sample preparation are given in Ref. [108].

Magnetic hysteresis at room temperature with field perpendicular to the film plane was measured using a Quantum Design physical properties measurement system (PPMS) with vibrating-sample magnetometer (VSM) in a maximum field of 3 T. The local magnetization distribution was studied using a digital instrument Dimension 3100 atomic force microscope with higher lateral resolution of 1–5 nm.

Figure 2.11 shows the magnetization curve of the [Co/Pt]/Ru multilayer as a function of the applied field, which is oriented perpendicular to the film plane. At zero applied field the film is in a nearly demagnetized state with a remanence close to zero. By increasing the field value, the magnetization increases almost linearly until it reaches saturation. At this point, the film is completely magnetized in the direction of the applied field. Reducing the field from positive saturation, the magnetization curve shows a kink in the first quadrant and, after that, abruptly reduces and continues to decrease with decreasing field. As the film is always parallel to the anisotropy axis, rotational processes are not expected in the magnetization reversal, thus the reversal may happen via nucleation of reversed domains and their expansion until at large negative field the former positive domains are fully expelled from the film. From these out-of-plane measurements the value for saturation polarization \(4\pi M_s = 0.765\) T and the coercive field \(\mu_0 H_c = 0.01\) T have been extracted [II].

Shown in Figure 2.12 are MFM images of the sample taken in three different remanent states. The dark and bright contrast corresponds to domains with magnetization pointing out and into the plane of the film, respectively. In the as-prepared state (Fig.2.12 (a)), the image is characterized by a labyrinth stripe domain pattern. After out of plane saturation, the multilayers exhibit a random maze domain pattern as shown in the MFM image in Fig.2.12 (b). The average domain width in both cases is about 180 nm. Applying a saturating in-plane magnetic field with subsequent in-plane AC demagnetization changes significantly the domain pattern, and also the average domain width is reduced to about 135 nm. The in-
plane magnetic field couples to the in-plane magnetization component of the domain wall and aligns the stripe domains parallel to the external field direction as it is visible in Fig.2.12 (c) [2]. Numerical calculations of dipolar sums predict that the parallel stripes are energetically favored over a labyrinth domain or maze domain structure (Fig.2.12 (a,b)) [111]. Above comparison, however, shows that the domain configuration in the remanent state depends strongly on the magnetic history, and the energetically lowest state has to be initiated by an appropriate demagnetizing procedure.

Figure 2.13 shows a series of MFM images for different magnetic fields applied perpendicular to the sample during measurement. The equilibrium domain width of the structured sample is comparable to that of the extended film shown in Fig. 2.12 a. For the given structure geometry and size we thus do not observe any influence of the lateral confinement on the domain configuration.

Figure 2.14: (Color) Sequence of MFM images recorded on the decreasing branch of a minor loop [II].
Starting from the demagnetizing state, in fields which are small compared to the saturation field, the domains change very slowly. The first magnetizing process can be observed at the rim of the element where bright domains oriented parallel to element edge disappear first. This can be understood from the unfavorably large magnetostatic energy for such oriented domains due to large stray fields [112]. By increasing the external magnetic field, the domains which are aligned parallel to the field grow while the oppositely aligned domains get smaller. This process occurs gradually, until the domains transform into isolated stripes and, in the end, into a bubble domain structure at higher fields. However, near the strip-out instability field (the field in which the elongated domains with opposite magnetization transform into bubble domains), a rapid growth of the preferably aligned domains can be observed. As the external field is increased, the domains contract so that only bubbles exist at magnetic fields greater than the strip-out field. Further increase of the external field causes the bubbles to shrink until at a critical field (collapse field) of 0.52 T they collapse (not shown here).

In order to obtain a more precise quantitative field value at which the strip-out instability occurs, a series of MFM images was recorded on the decreasing branch of a minor loop in a narrow field range close to saturation (Fig.2.14). By this we can ensure that the strip-out starts from isolated bubbles and is not influenced by the domain configuration at lower fields. Starting from highest value where the bubbles still exist (0.5 T), upon decreasing the field the bubble shape and configuration stay stable but the MFM contrast arising from the bubbles changes gradually down to 0.41 T. This is best seen in exemplary MFM profiles extracted from the measurements across an individual, isolated bubble as a function of the applied field (Fig.2.15).

The observed magnetization processes resemble those seen in single layer with perpendicular anisotropy, Sect. 1.7.3. Close to saturation, the labyrinth stripe domains contract to form isolated stripe domains and transform further into bubble domains. The bubble domains transform into elongated stripes at \( H_{bs} \) and collapse at a critical field \( H_{bc} \).

The calculation of the equilibrium parameters in \([\text{Co/Pt}]/\text{Ru}\) have been carried out with the characteristic length \( l_c = 4.43 \text{ nm} \). This value has been derived from analysis of the domain periods measured in series of \([\text{Co}(4\text{ Å})/\text{Pt}(7\text{ Å})]_X\) multilayers (see Fig. 2.5 and related text).
Figure 2.16: (Color) The characteristic functions $S_{bc}(d)$ and $S_{bs}(d)$ determine bubble sizes at collapse ($d_{bc}$) and strip-out ($d_{bs}$) fields. For [Co/Pt]/Ru multilayer ($l_c = 4.43$ nm) we obtained $d_{bc} = 69.76$ nm and $d_{bs} = 193.47$ nm. The inset shows the equilibrium values of the bubble diameters as a function of the bias field.

At zero field from the Eq. (2.10) the equilibrium stripe domain period $D_0 = 264.3$ nm for stripe period, which is very close to the observed stripe period after in-plane AC demagnetization (270 nm, Fig.2.12 (c)).

The stability functions of isolated bubble domain in the studied multilayer are given in Fig.2.16 in dependence of the diameter $d$ and their intersection with $l_c$ results in strip-out and collapse diameter. The inset shows the variation of $d$ within the stability region as a function of the applied field.

For the present [Co/Pt]/Ru multilayers we estimate $\mu_0 H_{bs} = 0.38$ T and $\mu_0 H_{bc} = 0.47$ T, values which are in good agreement with the experimental ones $\mu_0 H_{bs,exp} = 0.4$ T and $\mu_0 H_{bc,exp} = 0.52$ T.

In conclusion, the magnetization proceeds as typical for single layer thin films with perpendicular anisotropy via gradual growth of the domains oriented in the same direction with the applied field and contraction of those oriented opposite to the applied field. With only one free parameter, the characteristic length $l_c$, which was determined to 4.43 nm, the theory can describe the equilibrium domain width of the sample in the AC demagnetized state and the strip-out and collapse field of the bubble domains with very good precision. Combining this theory with domain observation thus helps to deduce physical properties such as wall energy and coupling interaction in complex magnetic multilayers.
2 Magnetic Multilayer With Interlayer Exchange Coupling

2.2 Instability of Stripe Domains in Multilayers With Antiferromagnetic Interlayer Exchange Coupling

In this section we describe our approach for equilibrium states in antiferromagnetically coupled multilayers. So far, theoretical analysis of magnetization states and processes in antiferromagnetically coupled multilayers with out-of-plane magnetization has been based on micromagnetic models of stripe domains, where the domain walls throughout the whole stack of the ferromagnetic layers sit exactly on top of each other [7]. In this chapter we show that this assumption is wrong. The antiferromagnetic interlayer coupling causes a lateral shift of the domain walls in the adjacent ferromagnetic layers.

In the first subsection we introduce a phenomenological theory of these complex stripe states. The analytical evaluation of a basic two-layer model shows that the formation and evolution of such “shifted” multidomain phases should appreciably influence the appearance and the magnetization processes of stripe states in perpendicular, antiferromagnetically coupled multilayers, see Ref. [III].

In the second subsection we discuss peculiarities of the ground state for multilayer with odd number of AF coupled ferromagnetic layers or ferroblocks, see Ref. [IV].

Finally we give theoretical explanation for magnetization reversal processes in AF coupled multilayers and compare our results with experiments on [Co/Pt]/Ru multilayers, see Ref. [VII].

2.2.1 Phase Diagram of Equilibrium States in Exchange Coupled Ferromagnetic Multilayers

As a model we consider stripe domains in a superlattice composed of \( N \) identical ferroblocks with \( X \) ferromagnetic layers of thickness \( t \) and \( X - 1 \) interlayers of thickness \( s \). Adjacent ferroblocks are antiferromagnetically coupled via a spacer of thickness \( S \). To simplify the solution of the magnetostatic problem at the first step, let us consider each ferroblock as a single ferromagnetic layer. In Sect. 2.1.1 we have shown that although this model does not converge to the rigorous solution, it gives a very small relative error in a wide range of geometrical and material parameters. In particular, for \([\text{Co}(4\text{Å})/\text{Pt}(7\text{Å})]_X\) multilayer we find that characteristic length \( l_c \) is about 4.4 nm (see Fig. 2.5 and related text). Thereby the ratio \( t/l_c \) for these multilayer is less then 0.1 and lies in the range where, for \( X \geq 5 \), the relative...
error of the single layer model is always less then 2% (see Fig. 2.9 and related text). In accordance with the above, we will consider multilayers with \( N \) magnetic layers of thickness \( T = X t + (X - 1)s \) and \( N - 1 \) nonmagnetic layers of thickness \( S \). The saturation magnetization and domain wall energy density averaged over ferro block are \( M_s^* = M_s \cdot X / [X + (X - 1)s/t] \) and \( \gamma_w^* = \gamma_w \cdot X / [X + (X - 1)s/t] \) respectively.

The stripe domain phase consists of domains with alternate magnetization \( M \) along the \( z \)-axis perpendicular to the multilayer plane. The domains in adjacent layers are laterally shifted along \( x \)-axis (Fig. 2.17). The reduced magnetic energy density \( e_{tot} = E_{tot} / (2\pi (M_s^*)^2TN) \) of the model can be written as a function of reduced parameters \( p = 2\pi T/D \), \( m = (d_+ - d_-) / D \), and \( u = 2a / D \), where \( T \) is the total thickness of magnetic layers (or ferromagnetic blocks),

\[
e_{tot} = \frac{2pL_c}{\pi T} - H M_s m + (1 - 1/N) \frac{\delta}{T} f(u, m) + e_m(p, m, u). \tag{2.39}
\]

The first term in (2.39) describes the domain wall energy, the second one is Zeeman energy, \( \delta = J / (2\pi (M_s^*)^2) \), \( J \) is the constant of interlayer exchange coupling \( (J > 0 \text{ – antiferromagnetic}, J < 0 \text{ – ferromagnetic}) \), and \( e_m \) is the stray field energy. Function \( f(u, m) \) in Eq. (2.39) is a piece-wise continuous function given by

\[
f(u, m) = \begin{cases} 
1 - 2|u| & \text{for } |u| \leq 1 - |m| \\
2|m| - 1 & \text{otherwise.} \tag{2.40}
\end{cases}
\]

The stray field energy \( e_m \) composed of the sum of self magnetostatic energy of each layer \( e_{m, s} \) (see Eq. (1.46)) and their interaction energy \( e_i \) which is derived by solving the corresponding magnetostatic problem similar to the one in the previous Sect. 2.1. In according to Eq. (1.16) the interaction energy between two magnetic blocks at the distance \( \omega T \) between their centres \( (\omega = S/T + 1) \) can be written as

\[
e_{i,2} = \frac{1}{2} \int_0^D \sigma_s(x) \left[ \phi_s(x-a, \omega T) + \phi_s(x-a, \omega T) - \phi_s(x-a, \omega T - T) - \phi_s(x-a, \omega T + T) \right] dx, \tag{2.41}
\]

where surface charge distribution for stripe domain pattern \( \sigma_s(x) \) and magnetostatic potential \( \phi_s(x, z) \) are given with Eqs. (1.40) and (1.44) respectively. After integration and simple transformation similar to the method above (see Sect. 2.1.1) the interaction energy (2.41) takes the form

\[
e_{i,2} = 2\pi M_s^2 t \frac{4}{\pi^2 p} \sum_{n=1}^{\infty} \cos \left( \frac{2\pi n a}{D} \right) \{1 - (-1)^n \cos (\pi nm)\} \left[ 2e^{-\omega p} - e^{-n(\omega-1)p} - e^{-n(\omega+1)p} \right]. \tag{2.42}
\]

In Eq. (2.42) we used the equality

\[
\int_0^D \cos (2\pi k x) \cos (2\pi n(x-a)) dx = \begin{cases} 
D \cos (2\pi n a / D) / 2, & k = n; \\
0, & k \neq n.
\end{cases}
\]
Then the density of the total magnetostatic energy per one layer (or ferroblock) \(e_m = e_{m,s} + e_{i,N}\) of \(N\) antiferromagnetically coupled layers can be written as

\[
e_m = \Omega(p, m, 0, 1) - \frac{1}{N} \sum_{k=1}^{N-1} (N - k) \Xi_\Omega(\omega k)
\]

where

\[
\Xi_\Omega(\omega k) = -2\Omega(p, m, u_k, \omega k) + \Omega(p, m, u_k, \omega k - 1) + \Omega(p, m, u_k, \omega k + 1),
\]

\[
\Omega(p, m, u_k, \omega) = \omega \left( m^2 + \frac{4}{\pi^2 \omega p} \sum_{n=1}^{\infty} \frac{1 - (-1)^n \cos(\pi nm)}{n^3} \left[ 1 - \cos(\pi n u_k) e^{-\omega np} \right] \right).
\]

and \(u_k = \frac{1}{2} \cdot (-1)^k u\).

The first derivatives of the function (2.45) with respect to \(m\), \(p\) and \(u\) have the following form:

\[
\frac{\partial}{\partial m} \Omega(p, m, u_k, \omega) = 2\omega m + \frac{4}{\pi^2 \omega p} \sum_{n=1}^{\infty} \frac{1 - (-1)^n \sin(\pi nm)}{n^2} \left[ 1 - \cos(\pi n u) e^{-\omega np} \right],
\]

\[
\frac{\partial}{\partial p} \Omega(p, m, u_k, \omega) = \frac{4}{\pi^2 \omega p} \sum_{n=1}^{\infty} \left( 1 - (-1)^n \cos(\pi nm) \right) \times \left[ e^{-\omega np} \cos(\pi n u) - 1 + \frac{\omega p e^{-\omega np} \cos(\pi n u)}{n^2} \right],
\]

\[
\frac{\partial}{\partial u} \Omega(p, m, u_k, \omega) = \frac{4}{\pi^2 \omega p} \sum_{n=1}^{\infty} \frac{1 - (-1)^n \cos(\pi nm)}{n^2} \left[ \sin(\pi n u) e^{-\omega np} \right].
\]

Minimization of the total energy (2.39) with respect to the parameter \(m\) gives the equation:

\[
HM_s = \left( 1 - \frac{1}{N} \right) \frac{\delta}{T \delta m} f(u, m) + \frac{\partial}{\partial m} \Omega(p, m, 0, 1) - \frac{1}{N} \sum_{k=1}^{N-1} (N - k) \Xi_\Omega(\omega k)
\]

where

\[
\frac{\partial}{\partial m} f(u, m) = \begin{cases} 0 & \text{for } |u| \leq 1 - |m|, \\ 2 & \text{otherwise}, \end{cases}
\]

and \(\partial \Omega(p, m, u, \omega)/\partial m\) is defined with Eq. (2.46). From Eq. (2.49) it follows that \(m \equiv 0\) at \(H = 0\). Thereby, the ground state of the system at zero external field is defined by two equilibrium parameters \(p\) and \(u\) only.

In recently investigated perpendicular AF multilayers \(D\) is of the same order as the ferroblock thickness \(T\) or exceeds it \([7, 19, 20]\). Because \(p \leq 1\), the convergence of the sum in Eq. (2.45) is very weak. Dipolar interactions between different surface charges or remote poles, as described by terms \(\propto \exp(-n \omega p)\) and those with large \(n\), have a sizeable effect for
wide stripes $D > T$. This requires a mathematically rigorous treatment of the infinite sum in (2.45). The summation of $\Omega$ by a method introduced above in previous sections reduces the stray field energy into an integral expression.

Using the relationship between trigonometric and hyperbolic functions for the expression in square brackets in (2.45) we get

$$1 - \cos(\pi nu) \exp(-\omega np) = \frac{1}{2} [1 - \exp(-\alpha⁺ np)] + \frac{1}{2} [1 - \exp(-\alpha⁻ np)]$$

where $\alpha ± = \omega ± i\pi u/p$ and $i$ – imaginary unit. Then according to identity of Eqs.(2.4) and (2.5) we can write:

$$\Omega(p, m, u, \omega) = \omega (1-u) - \frac{\alpha⁺² p}{\pi²} \int_0^1 (1 - \xi) \ln \left[ 1 + \frac{\cos²(\pi m/2)}{\sinh²(\alpha⁺ p\xi/2)} \right] d\xi$$

$$- \frac{\alpha⁻² p}{\pi²} \int_0^1 (1 - \xi) \ln \left[ 1 + \frac{\cos²(\pi m/2)}{\sinh²(\alpha⁻ p\xi/2)} \right] d\xi.$$ (2.51)

The function $\Omega(p, m, u, \omega)$ in the form (2.51) is a function of complex variables, but it is easy to show that values of the function are real ($\text{Im}(\Omega(p, m, u, \omega)) \equiv 0$). After some transformations the function $\Omega(p, m, u, \omega)$ can be written as a function of real variables only:

$$\Omega(p, m, u, \omega) = \omega (1-u) - \left( \frac{\omega² p}{\pi²} - \frac{u²}{p} \right) \int_0^1 (1 - \xi) \ln \left[ 1 + \frac{\cos²(\pi m/2) - G(\omega p\xi, \pi u\xi)}{F(\omega p\xi, \pi u\xi, \pi m/2)} \right] d\xi$$

$$+ \frac{4\omega u}{\pi} \int_0^1 (1 - \xi) \frac{2F(\omega p\xi, \pi u\xi, \pi m/2) - G(\omega p\xi, \pi u\xi)}{\sin(\pi u\xi) \sinh(\omega p\xi)} d\xi,$$ (2.52)

where

$$F(a, b, c) = \left( \frac{\sinh(a) + \sin(b)}{2\cos(c)} \right)^2, \quad G(a, b) = 1 - \cosh(a) \cos(b).$$

This integral representation of the stray field term enables an easy evaluation and minimization of the micromagnetic energy as well as the analysis of limiting cases. In particular, it is easy to see that for $u = 0$ the magnetostatic energy Eq. (2.52) completely coincides with the case of unshifted ferro stripes Eq. (2.5).

For the case of zero applied field $H = 0$ and compensated magnetization $m \equiv 0$ the magnetostatic energy can be significantly simplified. In particular for the function $\Omega_0(p, u, \omega) = \Omega(p, m, u, \omega)|_{m=0}$ one can get

$$\Omega_0(p, u, \omega) = 4 \left( \frac{\omega² p}{\pi²} - \frac{u²}{p} \right) I₁(\omega p, u_k) + \frac{8\omega u_k}{\pi} I₂(\omega p, u_k),$$ (2.53)
Figure 2.18: (Color) The magnetic phase diagram of states in reduced variables for layer thickness $T/L_c$ and interlayer exchange $\delta/L_c$ for $N = 2$ and $S/T = 0.1$. Thick lines indicate the first-order transitions from the shifted ferro stripe phase (b) into the antiferro stripe (d) ($\kappa - \gamma$) and homogeneous (c) phases($\alpha - \kappa$). These critical lines meet in a special critical point $\kappa(T_\kappa = 2.0816L_c, \delta_\kappa = 0.6385L_c)$, where a continuous transition line ends on a first-order line. The thin solid line $\alpha - \beta$ indicates the stability limit of the shifted ferro stripe phase. The dotted line ($\delta = 0$) indicates the second-order transition from the shifted stripes ($a > 0$) in antiferromagnetically coupled systems ($\delta > 0$) into the ferro stripes ($a = 0$) in ferromagnetically coupled or decoupled multilayers ($\delta \leq 0$). At the dashed line $T \equiv T_\kappa$ the antiferro phase (d) continuously transforms into the homogeneous antiferromagnetic states (c) for $\delta > \delta_\kappa$. Detail see in the text.

Thus, at $H = 0$ minimization of Eq. (2.39) with respect to $p$ and $u$ yields the equilibrium geometrical parameters, $D$ and $a$, for the stripe domains. Generally the solutions for the equilibrium multidomain states depend on the four control parameters $T/L_c$, $\delta/L_c$, $S/T$, and $N$ in the model. The phase diagram in variables $(T/L_c, \delta/L_c)$ for $S/T = 0.1$ and $N = 2$ plotted in Fig. 2.18 demonstrates the main features of these solutions and contain all possible ground states. Depending on the values of the materials parameters one of the four ground states is realized in the system: the shifted ferro stripe phase (a), the ferro (b) or antiferro (d) stripes, and the antiferromagnetic single domain state (c). The analytical results
Figure 2.19: (Color) Difference between the energies of the shifted ferrostripes and single domain AF phases $\Delta e_{\text{tot}}$ (in $2\pi M_s^2$ units) as functions of reduced shift $u$ along the lines $T/L_c = 1$ (a) and $3$ (b) indicated by labels (1) - (4) in Fig. 2.18. Stable or metastable shifted ferro stripes correspond to the potential wells marked by points in the energy profiles (1)-(4). The potential well for $u =1$ corresponds to the AF single domain state (a) and the unshifted AF stripes (b).

Presented in Figs. 2.18 and 2.20 exemplify a fundamental difference between multidomain states in antiferromagnetically coupled superlattices ($\delta > 0$) and those in multilayers with a ferromagnetic interlayer exchange ($\delta < 0$) or in decoupled multilayers ($\delta = 0$). In the latter cases $\delta \leq 0$ the ferro stripe phase is the ground state for arbitrary values of the control parameters $\delta, L_c, T, S$, and $N$. In particular, the rigorous solution revealed that for $\delta \leq 0$ parameters $u \equiv 0$ and $m \equiv 0$. Thereby the interlayer exchange term in the total energy (2.39) become a constant and parameters of domain structure are defined with the same equation as for decoupled multilayer (see Sect. 2.1.1).

In the antiferromagnetic case the ferro stripes can exist as stable or metastable state (i) only in a certain range of the control parameters (in Fig. 2.18 below lability line $\alpha - \beta$), and (ii) the ferro stripes are unstable with respect to lateral shifts of domain walls in the adjacent layers. Thus, in antiferromagnetically coupled superlattices stripe domains with parallel arrangement in the adjacent layers only exist as shifted ferro stripes. At the critical line $\alpha - \kappa - \gamma$ this phase transforms into the homogeneous state (with $T < T_\kappa = 2.0816 L_c$ for $N = 2$) or an antiferro stripes phase ($T > T_\kappa$) by a first-order transition. Below we give an analytical expression for the critical thickness $T_\kappa$ (see Eq. (2.67)). The shifted ferro stripes mode is the ground state of the system between the transition lines $\alpha - \kappa - \gamma$ and $\delta = 0$. Figure 2.19 illustrates the change in the energy balance of the system during the transition between ferromagnetic and AF states for the cases of fixed thickness of ferromagnetic layer indicated by two thin lines ($T/L_c = 1$ and $3$) in phase diagram Fig. 2.18.

Stripe domains with antiparallel arrangement across the stack, AF stripes, form equilibrium phases only for thicker layers, $T > T_\tau$, and for sufficiently strong AF coupling $\delta > \delta_\tau$ (Fig. 2.18 area (d)). In [Co/Pt]Ru multilayers with $J=0.45$ erg/cm$^2$, $M_s^* = 700$ emu/cm$^3$, and $\gamma_s^* = 8.9$ erg/cm$^2$ ($M_s^*$ and $\gamma_s^*$ are the values averaged over whole multilayer) [7]. This gives $L_c = 14.5$ nm and $\delta/L_c = 2J/\gamma_s^* = 0.1$. For [Co/Pt]$_3$NiO [Co/Pt]$_3$ with the largest values of $J = 0.044$ erg/cm$^2$ [19] the value of $\delta$ is about one order of magnitude smaller than in [Co/Pt]Ru systems. These values for $\delta$ are much smaller than the critical value $\delta_\tau$, and, thus, in [Co/Pt]Ru and [Co/Pt]$_3$NiO [Co/Pt]$_3$ systems single or multidomain states with parallel arrangement in
Figure 2.20: (Color) The equilibrium values of the reduced shift $2a/D$ and reduced period (inset) as functions of the reduced thickness $T/L_c$ for different values of $\delta = J/(2\pi M_s^2)$ in a two-layer ($N = 2$) with $\nu = 0.1$. Hollow points indicate the solutions at the transition line $\alpha - \kappa - \gamma$, and solid points show the shift at the lability line $\alpha - \beta$ in Fig. 2.18.

Adjacent layers, ferro stripes, form the the ground state (area (c) and (b) in Fig. 2.18). This is agreement with the experimental observations [7,20].

AF stripes are always stable with respect to a lateral shift. In the case $N = 2$, the antiferro stripe phase transforms into the homogeneous phase by the unlimited expansion of the stripe period at the critical line $T \equiv T_\kappa$ down to the critical point at $\delta_\kappa = 0.6385 L_c$. Below the transition line $\kappa - \gamma$ the antiferro stripes (with zero shift) still exist as metastable states down to the line $\delta \equiv 0$.

On the contrary to AF stripes, the solutions for the ferrostripe phase for $\delta > 0$ always has a finite shift (see Fig. 2.20). As shown in Fig. 2.20, the “exchange shift” $a$ can attain sizeable values only for a large value of exchange constant and strictly speaking should be considered as complex domain wall. Nevertheless it appreciably influences the appearance and stability of the ferro stripe phase.

The results above are supported by analytical and simple physical arguments, see Ref. [III]. Indeed, note that in antiferromagnetically coupled superlattices the exchange energy of the shifted ferro stripes includes a negative contribution linear with respect to $a$ (2.39). This is the mathematical reason for the instability of the solutions with zero shift. To elucidate this phenomenon we consider small shift distortions in an isolated domain wall. The perturbation energy (per domain wall length) $\Delta E(a) = \pm 2\pi M_s^2 e(a)$ can be written as a series with respect to the small parameter $a \ll s$ (the upper (lower) sign corresponds to (anti)ferro stripe states)

$$e(a) = -4\delta |a| + A(\nu)a^2 - B(\nu)s^{-2}a^4,$$

where $\nu = S/T$, $A(\nu) = (2/\pi)\ln[(\nu + 1)^2/(\nu(\nu + 2))]$, $B(\nu) = (2 + 6\nu + 3\nu^2)/[3\pi(\nu + 1)^2(\nu + 2)^2]$. The interlayer exchange coupling energy in Eq. (2.56) is linear with respect to the shift $a$ and negative in the case of the ferro stripes. This energy contribution yields solutions with finite $a = 2\delta A^{-1}(\nu)$ for arbitrary strengths of the antiferromagnetic exchange. On the contrary, for the antiferro stripes this energy is positive and the solution with zero shift remains stable.
In Ref. [7], experimental domain observations are reported on antiferromagnetic [Co/Pt]/Ru multilayers with $N = 2$ to 10, the magnetization $M = 700$ emu, interlayer exchange $J = 0.45$ erg/cm$^2$, and the parameter $\nu$ in the a range from 0.072 to 0.6. For these systems, the values of the shift $a$ vary from $a = 4.5$ nm ($\nu = 0.72$) to $a = 18.5$ nm ($\nu = 0.6$). These shifts amount to noticeable parts of the domain size, $D/2 \simeq 130$ nm [7]). The mathematical connection between values of the shift $a$ and the exchange constant $J$ can be used for an experimental determination of the strength of the antiferromagnetic coupling.

In Fig. 2.21 the dependence of equilibrium parameter $p$ on the thickness of magnetic layer for the shifted ferro stripes is shown for different values of $\delta/L_c$. The figure illustrates a monotonic character of the dependence $p(\frac{T}{L_c})$ along the transition line (green-red line) as well as along the lability line (black line). This fact can be used to derive a simple semi-analytical equation for the lability line. Indeed, using the expression for the total energy Eq. 2.25 the lability line for ferro stripes is defined by the system of equations:

$$\begin{align*}
\frac{\partial e_{\text{tot}}}{\partial p} &= 0, \\
\frac{\partial e_{\text{tot}}}{\partial u} &= 0, \\
\frac{\partial^2 e_{\text{tot}}}{\partial u^2} &= 0.
\end{align*}$$

Using the system of equations (2.57) one can write the equation for the lability line at $H = 0$ in parametric form:

$$\begin{align*}
\frac{T}{L_c} &= \frac{\pi}{2} \left[ \frac{\partial}{\partial p} \Omega(p, 0, 0, 1) - \frac{1}{N} \sum_{k=1}^{N-1} (N-k) \Xi \frac{\partial}{\partial p} \Omega(\omega k) \right]^{-1}, \\
\frac{\delta}{T} &= \frac{N}{2(N-1)} \left[ \frac{\partial}{\partial u} \Omega(p, 0, 0, 1) - \frac{1}{N} \sum_{k=1}^{N-1} (N-k) \Xi \frac{\partial}{\partial u} \Omega(\omega k) \right],
\end{align*}$$

where

$$\Xi \frac{\partial}{\partial p} \Omega(\omega k) = -2 \frac{\partial}{\partial p} \Omega(p, 0, u_k(p), \omega k) + \frac{\partial}{\partial p} \Omega(p, 0, u_k(p), \omega k-1) + \frac{\partial}{\partial p} \Omega(p, 0, u_k(p), \omega k+1),$$

$\partial \Omega(p, m, u, \omega)/\partial p$ and $\partial \Omega(p, m, u, \omega)/\partial u$ are defined with Eqs. (2.47) and (2.48) respectively, $u_k(p) = \frac{1-(-1)^k}{2} u(p)$ and $u(p)$ is established via following parametric equation

$$\frac{\partial^2 e_{\text{tot}}}{\partial u^2} = \frac{1}{N} \sum_{k=1}^{N-1} (N-k) \Xi \frac{\partial}{\partial u^2} \Omega(\omega k) = 0,$$

$$\frac{\partial^2}{\partial u^2} \Omega(p, m, u, \omega) = \frac{1}{p} \left[ \frac{\cosh^2(\omega p) + 2 \cosh(\omega p) \cos(\pi u) \cos(\pi m) + \cos^2(\pi m) - \sin^2(\pi u)}{(\cosh(\omega p) - \cos(\pi u))^2} \right].$$

Each value of the parameter, $p \in [0, +\infty)$, defines one point of the lability line. Equation (2.58) together with (2.59) define the lability line for arbitrary number $N$ and geometrical parameter $\omega = 1 + \nu = 1 + S/T$.

The system of equations which satisfy the conditions for the transition line between ferromagnetic and antiferromagnetic states is as follow:
Figure 2.21: (Color) The equilibrium values of the parameter \( p = 2\pi T/D \) as functions of the reduced thickness \( T/L_c \) for in a two-layer \((N = 2)\) with \( \nu = 0.1 \). Thick (black) line corresponds to the solutions at the lability line \((\alpha - \beta \) in Fig. 2.18) and thin green-red line corresponds to the solutions at the transition line \((\alpha - \kappa - \gamma \) in Fig. 2.18).

\[
\begin{align*}
\left\{ \begin{array}{l}
\varepsilon_{\text{tot(AF)}} &= \varepsilon_{\text{tot(F)}}, \\
\partial \varepsilon_{\text{tot(AF)}}/\partial p &= 0, \\
\partial \varepsilon_{\text{tot(F)}}/\partial p &= 0, \\
\partial \varepsilon_{\text{tot(F)}}/\partial u &= 0, \\
\partial^2 \varepsilon_{\text{tot(F)}}/\partial u^2 &> 0
\end{array} \right. \\
(2.60)
\end{align*}
\]

Here \( \varepsilon_{\text{tot(F)}} = \varepsilon_{\text{tot}} \) and defined with Eq. (2.39), \( \varepsilon_{\text{tot(AF)}} = \varepsilon_{\text{tot}}|_{u=1} \). Because of the complexity of equations in (2.60) this system of equation generally has to be solved numerically.

For \( T < T_\kappa \) metastable isolated domain walls can exist within the antiferromagnetically coupled ground state (Fig. 2.18(e)). Usually antiferromagnetic domain patterns are formed during demagnetization cycles in multilayers with the single domain antiferromagnetic ground state (phase (c) in Fig. 2.18), see, e.g., Refs. [6,7,19,20]. Contrary to regular antiferro stripes the solution for these isolated walls depending on the thickness of the magnetic layer exist with zero or nonzero shift and preserve their (local) stability down to vanishing antiferromagnetic coupling \( \delta = 0 \). The results above elucidate the nature of so-called “tiger-tail” patterns visible along these isolated domain walls of the antiferromagnetic phase [19,20] and recently observed as a “mixed state” of antiferro and ferro stripes [20]. Isolated domain walls within the homogeneous antiferromagnetic phase can play the role of nucleation centers for the ferro stripe phase. Within the metastability region of the shifted ferro stripe phase (area between \( \alpha - \beta \) and \( \alpha - \gamma \) lines in Fig. 2.18) sinusoidal distortions of antiferromagnetic domain walls transform into spin configurations corresponding to the ferro stripe phase. Such patterns have been reported in Ref. [20] and were called “tiger-tails”. During the first-order phase transition at the \( \alpha - \gamma \) line of the phase diagram, where the monodomain antiferromagnetic
phase and the shifted ferro stripes coexist, “tiger-tails” develop into ferro stripe patterns. The transformation of “tiger-tails” into extended areas with the ferro stripe phase was observed in Ref. [20, 108]. This is a “mixed state” composed of the homogeneous antiferromagnetic and the multidomain ferromagnetic phases. This particular evolution at a first-order transition, when magnetic phases nucleate within domain walls of competing phases have previously been observed in various bulk magnetic systems [109]. We give a detailed explanation of the stability region of isolated domain walls and “tiger-tails” in Chapter 2.4.

Phase diagram and general behavior of the antiferromagnetically coupled bilayer in the ground state which is given above are representative for any multilayer with even number \( N \) of ferromagnetic layers (ferromagnetic blocks). Peculiarities of the multilayer with odd number of magnetic layers as well as difference in ferro and antiferro stripes solution will be discussed in the following section.

Note that in Eq. (2.45) we have assumed that lateral shift of domain walls between all magnetic layer are equal. For \( N \geq 4 \) due to the so-called “exchange cut” (endmost layer has only one exchange coupled layer, while all internal layers have two neighbors) the lateral shift of domain walls in endmost and in internal layers should be different as is shown in inset (b), Fig. 2.22. This effect has close relation to the effect of non-coherent rotation of magnetization in neighboring magnetic layers which take place in antiferromagnetically coupled multilayer with in-plane anisotropy in applied magnetic field [9]. To elucidate the impact of this effect on ferromagnetic stripes solution we calculated phase diagram of equilibrium states.
Figure 2.23: (Color) The phase diagram of magnetic states in reduced variables for layer thickness $T/L_c$ and interlayer exchange $\delta/L_c$ for $N = 3$ and $S/T = 0.1$. Thick line indicate the first-order transitions from the shifted ferro stripe phase (b) into the antiferro stripe (c) ($\alpha - \gamma$). The thin solid line $\alpha - \beta$ indicates the stability limit of the shifted ferro stripe phase. The dotted line ($\delta = 0$) indicates the second-order transition from the shifted stripes ($a > 0$) in antiferromagnetically coupled systems ($\delta > 0$) into the ferro stripes (a) with $a = 0$ in ferromagnetically coupled or decoupled multilayers ($\delta \leq 0$). Details see in the text.

for multilayer with $N = 4$, Fig. 2.22. As follows from the figure with due account taken of exchange cut the position of transition line between ferro and antiferro stripe phases changes (solid green and red lines) in comparison with the model of equal shifts (dotted lines). However, for practically most important case of thin magnetic layer the transition line considering exchange cut effect (solid green line) and without it (dotted green line) converge with decreasing magnetic layer thickness. Thereby, in the case of thin magnetic layer, we can neglect the exchange cut (nonequal shifts) effect to define phase transition line and stability region of ferromagnetic stripes. It will be shown later in Sect. 2.3 that the exchange cut become significant for the description of magnetization reversal process in AF coupled multilayers with antiferromagnetic ground state.

2.2.2 Multilayers with odd number of magnetic layers

Now let us consider the case of odd number of magnetic layer. As a representative model we discuss multilayers with $N = 3$. Based on an analysis of the total energy (2.39) which is valid for any (even and odd) number of magnetic layer $N$, we have calculated the phase diagram of a three-layer system (Fig. 2.23). Above, we have discussed the case of multilayer with even number $N$ and showed that, with decreasing thickness of ferromagnetic layers, the antiferro stripe phase exhibits a second order transition to the monodomain state. As can
be seen from the phase diagram (Fig. 2.23), this transition is absent in the case of three layers (and generally in a system with an arbitrary odd number of layers). To clarify such a behavior of antiferro stripes with even and odd number of magnetic layers, it is helpful to use an expansion of the magnetostatic energy for the case of large domains. As the antiferro stripes are stable with respect to the lateral shift of domain walls \((u \equiv 1)\) using the Eqs. (2.39) and (2.43) one can write the total energy of the antiferro stripes in zero field \((H = 0, m = 0)\) in following form:

\[ e_{\text{tot(AF)}} = \frac{2pL_c}{\pi T} - (1 - 1/N) \frac{\delta}{T} + \Omega(p, 1) - \frac{1}{N} \sum_{k=1}^{N-1} (-1)^k (N - k) \Xi_\Omega(\omega k). \]  

(2.61)

where \(\Xi_\Omega(\omega k) = -2\Omega(p, \omega k) + \Omega(p, \omega k - 1) + \Omega(p, \omega k + 1)\), and expansion for function \(\Omega(p, \omega)\) in a series with respect to power of \(p\) is given by

\[ \Omega(\omega, p) = \frac{2\omega^2 p}{\pi^2} \int_0^1 \left(1 - \xi\right) \ln \left[1 + \frac{1}{\sinh^2(\omega p \xi/2)}\right] d\xi \approx \frac{2\omega^2 p}{\pi^2} \left(\frac{3}{2} - \ln \left[\frac{\omega p}{2}\right] + \frac{\omega^2 p^2}{72}\right) \]  

(2.62)

Then the minimization of the total energy (2.61) with respect to \(p\) \((de_{\text{tot, AF}}/dp = 0)\) gives the equation for equilibrium period. For even \(N\) this equation is given as

\[ \frac{\pi L_c}{T} = f_N(\tau) - \frac{P^2}{24} g_N(\tau), \]  

(2.63)

where \(f_N(\tau) = \sum_{k=1}^{N-1} (-1)^k [1 - k/N][2(\tau k)^2 \ln(\tau k) - (\tau k - 1)^2 \ln(\tau k - 1) - (\tau k + 1)^2 \ln(\tau k + 1)], g_N(\omega) = \sum_{k=1}^{N-1} (-1)^k (1 - k/N) [2(\omega k)^4 - (\omega k - 1)^4 - (\omega k + 1)^4],\) and \(\omega = \nu + 1 = S/T + 1,\) for even \(N\) one get \(g_N(\tau) = 3N\omega^2 + 1.\) Note that for systems with even number of ferromagnetic stacks the logarithmic term in Eq. (2.61) vanishes, and the equilibrium states are formed as a result of a competition between linear and cubic terms. For odd \(N\) the first non-vanishing term is the logarithmic term and one can neglect terms of higher order \((O(p^2))\)

\[ \frac{\pi L_c}{T} = \left(\frac{1}{2} - \ln \frac{p}{2}\right) \left(1 - \frac{N - 1}{N}\right) + f_N(\omega). \]  

(2.64)

Then the solutions for \(p\) are respectively

\[ p_e = \frac{2\pi T}{D_e} = \frac{24}{3N\omega^2 + 1} \sqrt{f_N(\omega) - \frac{\pi L_c}{T}}, \]  

(2.65)

for even \(N,\) and

\[ p_o = \frac{2\pi T}{D_o} = 2 \exp \left(\frac{1}{2} - N \frac{\pi L_c}{T} - f_N(\omega)\right), \]  

(2.66)

for odd \(N.\)

As follows from (2.65) and (2.66) the solutions for antiferro stripes in the case of even \(N\) exist only above critical thickness \((T_\kappa\) in Fig. 2.18 and 2.22) which is given by a simple analytical expression:

\[ T_{\text{cr}} = T_\kappa = \frac{\pi L_c}{f_N(\omega)}, \]  

(2.67)
Figure 2.24: (Color) The solutions $D/L_c (T/L_c)$ for ferro (F) and antiferro (AF) stripes in systems with $\nu = S/T = 0.1$ and different $N$. AF stripes exist for thickness larger than the critical thickness $T_{cr}$. For systems with even $N$ AF stripe period goes to infinity at the critical thickness. For systems with odd $N$ AF stripe exist for any thickness (Inset). The thin (green) line shows the dependence $D(T/L_c)/L_c$ for the period of stripes in a single layer.

while for odd $N$ solutions exist up to $T = 0$. The equilibrium period and existence ranges of typical solutions for ferro and antiferro stripes are shown in Fig. 2.24. The solutions for ferro stripes exist for arbitrary values of $T$ and $S$. For the two limiting cases of large and small spacer thickness the solutions asymptotically approach the behavior of the known solutions for individual layers (see Ref. [21, 110]) with a thickness $T$ for the case $S \gg T$ and with effective thickness $TN$ for $S \ll T$. The solutions for antiferro stripes with even $N$ exist only in an interval bounded from below, $T > T_{cr}(N)$. The period $D$ tends to infinity at a critical thickness $T_{cr}$. For odd $N$ antiferro stripes exist for arbitrarily small layer thickness $T$, similarly to ferro stripes. However, their periods increases so steeply that a single domain state is effectively reached, when the period exceeds the lateral size of the layer. The calculated periods for odd-numbered multilayers have similar size as those for antiferro stripes with even $N$ (see inset in Fig. 2.24).
2.3  Metamagnetic Transition in Antiferromagnetically Coupled Multilayers

In this section we discuss magnetization reversal processes in antiferromagnetically coupled multilayers. The magnetization processes in such multilayers strongly depend on the type of the ground state, which has been discussed in previous Section 2.2.1. In an applied magnetic field the FM stripe phase evolves similarly to common stripes in ferromagnetic layers [2]. In Sect. 2.1 we gave detailed theoretical description and experimental investigations of multidomain states and magnetization processes in [Co/Pt]/Ru multilayers with FM stripe ground state. Here we mainly concentrate on magnetization reversal process in [Co/Pt]/Ru multilayers with the AF single domain ground state. In this case, AF phases transform into the saturated state via a first order transition accompanied by the formation of multidomain states [VIII]. In some basic physical aspects the multilayers in the AF ground state are similar to bulk antiferromagnets with strong uniaxial anisotropy (metamagnets) [113] and, thus, can be classified as synthetic metamagnets, see Ref. [114] and Ref. [VIII]. In particular, characteristic magnetic-field-induced domains have been observed in [Co/Pt]/Ru [20,107] and [Co/Pd]/Ru [106] superlattices. They are analogous to metamagnetic domains which have been observed in many bulk metamagnets [115].

The close physical resemblance between the metamagnetic domains and common ferromagnetic stripes in multilayers has been clearly demonstrated in various experiments on perpendicular multilayers, Ref. [20]. E.g., it was shown that the magnetization curve of an antiferromagnetic \( \{[Co(4Å)/Pt(7Å)]_{10}/Co(4Å)/Ru(9Å)\} \) multilayer in the region of a bulk metamagnetic transition is similar to the magnetization curve of a pure ferromagnetic \( \{[Co(4Å)/Pt(7Å)]_5Co(4Å)/Pt(57Å)]_5 \) multilayer. For more detail, see Ref. [20], Fig. 30. Here, we give a theoretical description for the magnetic-field-driven evolution of the metamagnetic domains and compare it with experimental data. The comparison of theoretical calculations for the anhysteretic magnetization process and the possible magnetic equilibrium phase sequences allows to understand the experimentally observed magnetization processes in [Co/Pt]/Ru superlattices with AF single domain ground state.

2.3.1  Model and Theoretical Analysis

Here, as well as before, as a basic model for \( \{[Co/Pt]_{X-1}/Co/Ru\} \) superlattices we consider \( N \) identical “ferromagnetic blocks” composed of \( X \) bilayers \([Co(t)/Pt(s)]\), antiferromagnetically coupled via \( N-1 \) Ru(S) spacers (where \( t, s \) and \( S \) are the thicknesses of the corresponding layers, Fig. 2.25 a). Here we will consider the cases with an even number of \( N \) only. Due to their net magnetization multilayers with an odd number of \( N \) can be considered as artificial ferrimagnets. These systems are markedly different from the antiferromagnetic systems discussed here.

We assume that the perpendicular uniaxial anisotropy originating from the Co interface anisotropy is strong enough to stabilize the perpendicular orientation of the magnetization \( \mathbf{M} \) in magnetic domains (Fig. 2.25). In the antiferromagnetic single domain phase a sufficiently strong magnetic field applied perpendicular to the multilayer surface (a bias field) “overturns” the antiparallel magnetization (Fig. 2.25).

The reorientation field for a certain ferromagnetic block depends on the strength of the
Figure 2.25: (Color) a) Sketch of an antiferromagnetically coupled multilayer consisting of \{[Co(h)/Pt(t)]_{X-1}/Co(h)/Ru(s)\}_N with \(X = 3\) and \(N = 6\) where the surface layer has already switched and metamagnetic stripe are formed in the interior. b) The layers are equivalent to stripes in two ferromagnetic blocks separated by distance \(L = 2S + T\), where \(T = Xt + (X - 1)s\) is the ferroblock thickness.

antiferromagnetic interaction with adjacent blocks. For a multilayer consisting of identical blocks, antiferromagnetic coupling of the interior blocks is two times stronger than that of the two endmost layers. Correspondingly field-driven reorientations may occur as a two step process: the surface block having its magnetization oriented against the applied field switches before the remaining interior blocks reorient. Both processes in the surface layer and in the bulk of the multilayer stack usually proceed gradually by redistribution of stripe domains with alternate, up-down, orientation. In the following these domain states are referred to as metadomains or metamagnetic domains. Due to long range magnetostatic interactions the walls of the bulk metamagnetic domains sit exactly on top of each other in ferromagnetic layers throughout the whole stack, Fig. 2.25. Furthermore, in the infinite layer approximation homogeneously magnetized ferromagnetic blocks do not interact with other blocks due to the localization of their stray fields within the layers. Thus, such metamagnetic domains are described by the regular model of a two-phase domain structure in a ferromagnetic multilayer consisting of \(X\) equidistant ferromagnetic monolayers that form one ferroblock (Fig. 2.25 a).

In the case of the metadomains in the bulk of the multilayer stack, the spacers have different thicknesses, \(s\) within Co layers and \(L = 2S + T\) \((T = Xt + (X - 1)s\) ) between ferroblocks (Fig. 2.25 b).

The energy density of the metamagnetic stripes (per one magnetic layer) in an applied magnetic bias field \(H\), [VIII]

\[
E_{\text{tot}} = E_{w} + E_{\text{ex}} + E_{m} - HM_s(d_+ - d_-)/D,
\]

as before includes the domain wall energy \(E_w = 2\gamma_w/D\) (where \(\gamma_w\) is the area energy density, \(D = (d_+ + d_-)\) is the stripe period, and \(d_\pm\) are the domain sizes, Fig. 2.25); the interlayer exchange energy \(E_{\text{ex}}\), stray field energy \(E_{m}\), and Zeeman energy. Note that for metamagnetic domains the exchange coupling is proportional to the difference between domain sizes, \(E_{\text{ex}} \propto (d_+ - d_-)\) (Fig. 2.25 b). This means that the interlayer exchange acts as a certain applied field. Introducing an effective exchange bias field,

\[
H_{\text{ex}} = \alpha J/\langle tXM_s \rangle,
\]
Figure 2.26: (Color) The magnetic phase diagram of the equilibrium states for multilayer with \( N = 2 \), \( X = 1 \) in reduced variables for magnetic layer thickness \( t/l_c \) and bias field \( H/(4\pi M_s) \). Metamagnetic stripes exist within area \( c - g - d \). The thick line \( c - d \) indicates the first-order transition between metamagnetic and shifted ferro stripes. The shifted ferro stripes phase transforms discontinuously into the antiferromagnetic and ferromagnetic phases along lines \( b - c \) and \( d - e \), correspondingly. The first-order lines meet in the triple points \( c \) (0.874,0.043) and \( d \) (0.845, 0.072).

allows to include it into the Zeeman term. Here \( J \) is the strength of the antiferromagnetic interlayer coupling, and the factor \( \alpha = 1 \) or 2 has to be used for surface and internal metamagnetic domains, respectively. After that, the energy \( E_{\text{tot}} \) from Eq. (2.68) can be reduced to the following form

\[
E_{\text{tot}} = 8\pi M_s^2 l_c^2 D - H M_s m + 2\pi M_s^2 e_m
\] (2.70)

where \( H = (H - H_{ex}) \) is an effective magnetic field, \( l_c = \gamma w/(4\pi M_s^2) \) is the characteristic length, \( m = (d_+ - d_-)/D \) equals the reduced magnetization of the stripe phase, and the magnetostatic energy density \( e_m = e_m(D, m) \) describes the stray field energy of the stripes and is a function of the variables \( D \) and \( m \). Everywhere in this section we use parameters of stripe domain period \( D \) and thicknesses \( t \), \( s \), and \( S \) in real units instead of reduced ones. This is useful for the following comparison between experimental and theoretical data given at the end of the section. The energy density (2.70) functionally coincides with that for a ferromagnetic multilayer in the applied field \( \mathcal{H} \) (compare with Eq. (2.8)). This reduces the problem to a model for common ferromagnetic domains in multilayers with spacers of thicknesses \( L = 2S + T \) in an applied magnetic field \( \mathcal{H} \) (Fig. 2.25).

Using the stripe domain model introduced in Sect. 2.1.1, the magnetostatic energy \( e_m \) can be written as

\[
e_m(D, m) = 1 - \Omega(t) + \Xi(t, s, S).
\] (2.71)
where
\[ \Omega (\omega) = \frac{4\omega^2}{\pi tD} \int_0^1 (1 - \xi) \ln \left[ 1 + \frac{\cos^2(\pi m/2)}{\sinh^2(\pi \omega \xi / D)} \right] d\xi. \]

Note that here \( \omega \) is the distance between charged planes in real units.

Let us remind that the function \( 1 - \Omega (t) \) is the stray field interaction between the planes within the same Co layers, while the energy \( \Xi_\Omega (t, s, S) \) describes interlayer interactions. To define this energy we introduce the following auxiliary functions (for detail see Sect. 2.1.1)
\[
\Xi_F (t, s, S) = \sum_{k=1}^{nX-1} \sum_{j=k+1}^{nX} \frac{2F(\omega_{kj} - t) - F(\omega_{kj} + t)}{nX},
\]
with \( \omega_{kj} = \sum_{i=k}^{j-1} [t + s_i] \) and
\[
s_i = \begin{cases} 
2S + T & \text{for } i = mX, \quad m = 1 \ldots n \\
S & \text{otherwise.}
\end{cases}
\]

In Eq. (2.72) \( n \) is the number of ferromagnetic \([\text{Co/Pt}]_{X-1}/\text{Co}\) subblocks involved in the metamagnetic transition. \( n=1 \) and \( N/2 - 1 \) respectively for surface and bulk metamdomains. The stray-field energy Eq. (2.71) is the generalization of the energy derived in Refs. [110], for the case of multilayers with different spacer thicknesses [III]. Such multilayers have, e.g., been treated in Ref. II.

The equilibrium domain sizes \( d_\pm \) are derived by minimization of the energy in Eq. (2.70) with respect to the domain period \( D \) and the imbalance of up and down domains \( m \). This leads to the following two coupled implicit equations,
\[
\Upsilon(t) - \Xi_\Upsilon(t, s, S) = \pi t t, \\
\Psi(t) - \Xi_\Psi(t, s, S) = (H - H_{ex})/(2M_s)
\]
with
\[
\Upsilon (\omega) = \omega^2 \int_0^1 \xi \ln \left[ 1 + \frac{\cos^2(\pi m/2)}{\sinh^2(\pi \omega \xi / D)} \right] d\xi
\]
and
\[
\Psi (\omega) = \frac{\omega^2 \sin (\pi m)}{tD} \int_0^1 \frac{(1 - \xi) d\xi}{\sinh^2(\pi \omega \xi / D) + \cos^2(\pi m/2)}.
\]

For the notation of function \( \Xi_\Upsilon \) and \( \Xi_\Psi \) see Eq. (2.72).

The transition fields \( H_{s1,s2} \) delimiting the existence regions for metamagnetic domains equal
\[
H_{s1,s2} = H_{ex} \mp H_s,
\]
where the critical field \( H_s \) is the transition field of stripe domains for ferromagnetic multilay-
ers, as derived from the equations
\[ H_s = M_s [\psi (h) - \Xi_\psi (x, s, S)] \] (2.75)
and
\[ \gamma (h) - \Xi_\gamma (h, s, S) = 2\pi l_c t \] (2.76)
with
\[ \psi (\omega) = 2\omega \arctg \left( \frac{\omega}{d_c} \right) - d_c \ln \left[ 1 + \left( \frac{\omega}{d_c} \right)^2 \right], \]
\[ \gamma (\omega) = d_c^2 \ln \left( 1 + \frac{\omega^2}{d_c^2} \right) + \omega^2 \ln \left( 1 + \frac{d_c^2}{\omega^2} \right). \]

The parameter \( d_c \) is equal to the width of the isolated stripe domain of the minority phase at the transition field \( H_s \), see Ref. [22] and Ref. I (for detail see Sect. 2.1.1, Model I). The functions \( \Xi_\psi \) and \( \Xi_\gamma \) are defined according to the Eq. (2.72).

To demonstrate main features of these solutions first let us consider a simple model for \( N = 2 \) (Fig. 2.26). In a magnetic field the antiferromagnetic phase transforms into the saturated state via a specific multidomain phase. This is similar to a metamagnetic phase transition in bulk antiferromagnets [113]. In an intermediate (metamagnetic) phase domains arise only in one of the ferromagnetic blocks while the other remains in the homogeneous (saturated) state (Fig. 2.26).

The metamagnetic transition has been observed in \([\text{Co/Pd}]/\text{Ru}\) multilayers with \( X = 7 \) and \( N = 2 \), \( t = 0.4 \text{ nm} \), \( s = 1.8 \text{ nm} \), and \( S = 0.8 \text{ nm} \) [106]. For this multilayer Eqs. (2.73) yields the domain period \( D_0 = 3.43 \mu\text{m} \) at \( H = H_{ex} \), and the width of this region \( \Delta H = 3 \text{ mT} \). For the experimental value \( H_{ex} = 0.126 \text{ T} \) and the saturation magnetization \( 4\pi M_s = 1.85 \text{ T} \) the equation (2.69) yields \( \delta = 0.38 \text{ nm} \). The calculated \( H_{s1} \), \( H_{s2} \) and \( H_{ex} \) using this value of the exchange length, for these \([\text{Co/Pd}]/\text{Ru}\) systems (Fig. 2.26, Inset) shows a widening of the metastable region for \( X > 7 \).

In the multilayers with \( N \geq 4 \) the metamagnetic transition occurs first in the surface layer at \( H \propto \delta/(tX) \), and then in the internal layers at higher field \( H \propto 2\delta/(tX) \). This kind of two-step transition in an external field has been observed in \([\text{Co/Pt}]/\text{Ru}\) systems [20]. Similarly, it also occurs in antiferromagnetically coupled multilayers with in-plane magnetization [9]. The typical magnetic phase diagram of the equilibrium states in reduced variables for layer thickness \( t/l_c \) and applied bias field \( H/4\pi M_s \) is shown in Fig. 2.27. It is calculated for a stack of \( N = 4 \) single ferromagnetic layers \( (X = 1) \) separated by an interlayer with reduced thickness \( S/l_c = 0.2 \), normalized exchange \( J/(2\pi M_s^2 l_c) = 0.2 \), and the assumption of constant characteristic length. Note that magnetic phase diagrams, Figs. 2.26 and 2.27, also approach the single layer model where each \([\text{Co/Pt}]_X \) ferromagnetic blocks are assumed to be single magnetic layers with reduced parameters of the thickness \( T = Xt + (X - 1)s \), characteristic length \( L_c = l_c [X + (X - 1)s/t]/X \) and saturation magnetization \( M_s = M_s \cdot X/[X + (X - 1)s/t] \) (for details see Sects. 2.1.1 and 2.2.1). This diagram holds six possible magnetic phases labeled as (I) – (VI). At zero field the ferromagnetic stripe domain state (VI) is more favorable for thick layers \( (t > t_o) \). The antiferromagnetic interlayer coupling causes a relative shift of
domains in adjacent layers when forming the ferromagnetic ground state. This leads to an instability of the ferrostripe solutions below the critical thickness $t_a$, where the homogeneous AF state (I) becomes more favorable, see Ref. [III]. For external fields parallel to the surface normal the transition between homogeneous AF state and the ferrostripes occurs along the line $a$-$b$ ($t_b < t_a$). Such transitions have been observed in [Co/Pt]/Ru multilayers with wedged Co layers [20]. For $t < t_b$ an increasing magnetic field induces three successive transitions. Critical lines $ob$ and $oc$ confine the region with surface metamagnetic domains (phase (II)). At the transition line $o$-$c$ the surface layer reaches the saturated state. The system remains in this ferrimagnetic state (III) up to the transition into the bulk (or internal) metamagnetic state (IV) (line $o'$$-d$). The two dashed lines in Fig. 2.27 are defined by Eq. (2.69). At these special lines the competing domains have the same sizes, $d^{(+)} = d^{(-)}(m = 0)$, and the total magnetization of the multilayer includes contributions only from the saturated layers. In particular for $N = 4$, the total magnetization at these lines equals $(1/4)M^s_0$ (surface domains) and $(3/4)M^s_0$ (bulk domains). $M^s_0 = M_sNX$ is defined as the saturation magnetization of a multilayer containing $NX$ cobalt layers. This allows to extract values of the exchange coupling from the magnetization curves. For $N = 4$ both the surface and bulk metamagnetic domains are formed only in one layer (Fig. 2.27). For multilayers with $N > 4$ the bulk metamagnetic transition involves more than one layer. Correspondingly, such transitions have wider existence regions than those for surface domains. Thus, for a larger number $N$ of ferromagnetic blocks, the regions with surface and bulk domains merge ($H_d < H_b$). In this case metamagnetic domains occur all along the stack similarly to domains in the ferrostripe phase (VI), however, only every second layer forms the stripes, as the intervening layers are already fully saturated. For these configurations $n = N/2$ and the parameter in the definition of the bias field, Eq. (2.69), is given by $\alpha = (2n - 1)/n$. Accordingly, for $H = H_{ex}$
The system is $N = 10$, $X = 5$. Inset (A) corresponds to sample A, inset (B) corresponds to sample B (in your notation). The value of interlayer exchange constant $J$ estimated from corresponding magnetization curves equals 0.391 erg/cm$^2$ and 0.544 erg/cm$^2$ respectively for the sample A and B. First I used sample B, you propose to use sample A. For both samples theoretical curves fit experimental data in a similar way. But probably you are right and the sample A looks more appropriate. Which of them we should include in the paper in your opinion?

Figure 2.28: (Color) SQUID hysteresis loop measured on $[\text{Co(4Å)/Pt(7Å)}]_{X-1}/\text{Co(4Å)/Ru(9Å)}_N$ multilayers with $X=5$ and $N=10$ (open symbols). Solid line correspond to the calculated magnetization curve for equilibrium states [VII].

the magnetization equals $(1/2)M_s^*$. 

2.3.2 Comparison Between Theoretical Model and Experimental Results

Below we give description of the experiment and comparison between experimental and theoretical data. The studied systems are $\{[\text{Co(4Å)/Pt(7Å)}]_{X-1}/\text{Co(4Å)/Ru(9Å)}\}_N$ multilayers with $N = 18$ and $X = 7, 8, 9$ and another multilayer with $N = 10$ and $X = 5$. Details on sample preparation are given in [20]. The sample series with $N = 18$ and varying $X$ has been chosen to test the predictions of Fig. 2.27, which suggest metamagnetic transitions for films with AF ground state (small Co/Pt thickness) and a simple FM behaviour for multilayers with larger Co/Pt thickness. Indeed, the zero-field domain structure was found to be ferromagnetic for samples with $X = 9$ [II] and antiferromagnetic for multilayers with $X = 7, 8$ [20]. The additional prediction of separated bulk and surface metamagnetic transitions is best studied in the multilayer with $N = 10$ and $X = 5$.

Magnetic hysteresis at room temperature, with field perpendicular to the film plane was measured using a Quantum Design physical properties measurement system with vibrating-sample magnetometer (VSM) or a superconducting quantum interference device (SQUID). In one sample ($N = 18$, $X = 8$) the evolution of the domain structure has been studied via in-field magnetic force microscopy (MFM) using a digital instrument Dimension 3100 atomic force microscope with MFM extender box for phase shift measurements.

Figures 2.28 and 2.29 show the hysteresis loops of the samples measured at 300 K with the magnetic field applied perpendicular to the sample surface. The magnetization curve for a $\{[\text{Co(4Å)/Pt(7Å)}]_4/\text{Co(4Å)/Ru(9Å)}\}_{10}$ multilayer with separate surface and bulk reversal steps (Fig. 2.28) clearly demonstrates the two-step character of the metamagnetic transition. The smaller steps at $\pm 0.12$ T result from the switching of the surface blocks at either the top or the bottom of the multilayer stack. The larger reversal steps occurring at higher fields
Figure 2.29: (Color) Hysteresis loop for $N = 18$ and $X = 8$ measured by VSM with the field perpendicular to the sample surface [VII]. Insets shows magnetization curves for $X = 7$ [108] and 9 [II]. Solid lines correspond to the theoretically calculated magnetization curve of equilibrium states.

$(H = \pm 0.32 \text{ T})$ are connected with the bulk metamagnetic transition. Such a distinct two-step reversal is expected for this multilayer since it has a small number of Co/Pt repeats $X$ per block.

Magnetization curves for the samples with $N = 18$ (Fig. 2.29) reveal an other possible scenario of metamagnetic transition. For $X = 8$, at small magnetic fields up to about 0.2 T, the sample displays a plateau region in which the magnetization of the [Co/Pt] blocks aligns in an AF configuration along the easy axis and is therefore less susceptible to the applied field. Increasing the field value above 0.2 T the antiferromagnetic coupling strength is exceeded which leads to a step-like increase of the magnetization. For higher fields, magnetization increases almost linearly until it reaches saturation. In the field region of 0.2 T to 0.6 T a mixed state of metamagnetic domains is expected which is studied in more detail by in-field MFM observation. A similar behaviour has been observed for a multilayer with $X = 7$ [108] while for $X = 9$, the system shows a purely ferromagnetic behaviour [II], as obvious from the absence of the plateau around remanence.

The calculation of equilibrium states of the system described by Eq. (2.70) can be done with reduced control parameters such as $H/(4\pi M_s)$, $J/(2\pi M_s^2)$, $t/l_c$, $s/l_c$, $S/l_c$, $N$ and $X$. However to define quantitative properties of the system one should determine the material parameters $J$, $M_s$, and $l_c$ in addition to the geometrical parameters.

From the magnetization curves for the samples with $N = 10$, $X = 5$ (Fig. 2.28) and $N = 18$, $X = 7$ (Fig. 2.29) we estimate the exchange field as the one corresponding to the center between up and down branches at $M/M_s^*$ equal to the middle of magnetization step. Theoretically the middle of magnetization steps corresponding to the bulk ($N = 10$, $X = 5$) and mixed ($N = 18$, $X = 7$) metamagnetic transition are equal $M/M_s^* = \frac{3}{5}$ and $= \frac{1}{2}$ respectively. Then in accordance with Eq. (2.69) we determine the interlayer exchange constants for these systems as $J = 0.391 \text{ erg/cm}^2$ for $N = 10$, $X = 5$ and 0.73 \text{ erg/cm}^2 for $N = 18$, $X = 7$. For $N = 10$, $X = 5$ we used the magnetization step corresponding to the bulk metamagnetic transition only. This choice is necessitated because of the strong hysteretic
Figure 2.30: (Color) Magnetic phase diagram with $X$ and a bias field $H$ as independent variables for the $[\text{Co/Pt}]_{N=10}/\text{Ru}$ multilayer with $N=10$ and $J=0.39$ erg/cm$^2$. For $X \geq 7$ the system has FM striped ground state. For thin ferroblocks $X \leq 6$ the regions with surface (II) and bulk (IV) metamagnetic domains are separated by the ferrimagnetic phase (III).

From the experimental measurements for the $\{[\text{Co}(4\,\text{Å})/\text{Pt}(7\,\text{Å})]_8/\text{Co}(4\,\text{Å})/\text{Ru}(9\,\text{Å})\}_{18}$ multilayer [II] we estimated saturation magnetization of individual cobalt layer as $M_s=1700$ emu/cm$^3$. We had estimated the value of $l_c=4.43$ nm as a best fit of the experimentally measured dependence of stripe domain period versus number of cobalt layers $X$ in a pure $[\text{Co/Pt}]_X$ multilayer, see Ref. [20] and [I] (see Fig. 2.5 and related text). The solid lines in figures 2.28 and 2.29 have been calculated as anhysteretic magnetization curves from Eq. (2.73) under the assumption that the average magnetization of the multilayer is

$$M = \begin{cases} \frac{1}{5}M_s^* \left[ \frac{1}{5} (1 + m_s) + \frac{4}{5} (1 + m_b) \right], & N = 10, \; X = 5; \\ \frac{1}{2}M_s^* [1 + m_m], & N = 18, \; X = 7 \text{ and } 8, \end{cases}$$

where subscripts $s$, $b$ and $m$ correspond to the surface, bulk and mixed type of metamagnetic transition respectively. It can be seen, that the general behaviour, i.e. the appearance or absence of bulk and surface metadomains and also the general approach to saturation is well described by the calculations, whereas the hysteretic experimental behaviour stemming from domain wall pinning can obviously not be explained by the anhysteretic model.

The phase diagrams Figs. 2.30 and 2.31 show existence regions for the magnetic states in the multilayer with fixed number $N$ and different values of $X$. These diagrams may be regarded as discretized analogues of Fig. 2.27. The thickness of $[\text{Co}(4\,\text{Å})/\text{Pt}(7\,\text{Å})]_{X-1}/\text{Co}(4\,\text{Å})$ block does not vary continuously, but in discrete steps with the number of cobalt layers $X$. Furthermore we take into account magnetostatic interaction between individual ferromagnetic layers and fix corresponding material parameters of each cobalt layer ($l_c=4.43$ nm, $M_s=1700$ emu/cm$^3$).

The phase diagrams display how the metamagnetic multidomain states change for the experimentally studied multilayers. As discussed with respect to Fig. 2.27, for large thickness...
Figure 2.31: (Color) Magnetic phase diagram with $X$ and a bias field $H$ as independent variables for the $[\text{Co/Pt}]/\text{Ru}$ multilayer with $N=18$ and $J=0.73 \text{ erg/cm}^2$. For $X \geq 9$ the system has FM striped ground state. For thin ferroblocks $X \leq 6$ the regions with surface (II) and bulk (IV) metamagnetic domains are separated by the ferrimagnetic phase (III). For $X = 7, 8$ metamagnetic domains are formed all along the stack.

of the ferromagnetic block, the multilayer remains in the ferrostripe regime (VI) for all applied fields until the sample saturates (V). For the studied systems this behaviour is expected for $X \geq 7$ ($N = 10$) and $X \geq 9$ ($N = 18$). Multilayers with smaller number of layers ($X \leq 6$ for both systems) show a two-step magnetization behaviour with a pronounced plateau at $M = 0$ for small fields until the surface layer switches through a surface metamagnetic transition in a small field range (II) as visible in Fig. 2.28 for $N = 10$ and $X = 5$. This is followed by a second plateau (III) that marks the existence range of the surface metadomains. Then follows the well separated bulk metamagnetic transition. Here, stripe domains exist alternatingly in internal layers of the multilayer stack, as sketched in Fig. 2.25 (A). For intermediate thickness ($X = 7, 8$) of $N = 18$ surface and bulk metamagnetic transition merge. Thus, only a single-step magnetization process is observed when the field overcomes the AF coupling. In those cases, the surface and internal metamagnetic domain structures take on a mixed appearance. These theoretical results are matched by the hysteresis measurements presented in Fig. 2.29 and by other experimental findings of Refs. [20] and [II].

The equilibrium metamagnetic stripe domain sizes for the multilayers with $N = 18$, $X = 8$ are plotted in Fig. 2.32. While $d^+$ and $d^-$ grow and shrink at the cost of each other, the domain period develops a minimum at $H = H_{\text{ex}}$. Particularly, we find that the smallest domain period, $D=352.6 \text{ nm}$ should be observed for $H = H_{\text{ex}}$ at a field of 0.253 T. The isolated stripe domains of the minority phase have the same width $d_c=101 \text{ nm}$ for $H = H_{s1}$ (0.104 T) and $H = H_{s2}$ (0.4 T) (see Fig. 2.32). The calculated magnetization curves for $N = 18$ and different values of $X$ (Fig. 2.32, Inset) reflect the different types of reorientation in the system. The same magnetization curves are plotted in Fig. 2.29 for comparison with the experimental data.

At the critical field $H_{s1}$ ($H_{s2}$) the metamagnetic stripes transform into the homogeneous antiferromagnetic phase (saturated ferrimagnet). During this transition the period extends to infinity while the size of the minority phase $d^{(-)}$ has a finite value at the saturating field.
Figure 2.32: (Color) The equilibrium values of domain sizes \( d_\pm \) and the stripe period \( D \) as functions of the applied field in the multilayer with \( N=18, X=8 \). The open circles denote the critical width of isolated stripe domains of minority phase \( d_c \) at critical fields \( H_{s1} \) and \( H_{s2} \). The inset shows the theoretical magnetization curves for \( X = 6, 7, 8 \) and 9.

As is shown in Fig. 2.33 the behaviour of equilibrium domains for bulk and surface metamagnetic transition for multilayers with \( N=10, X=5 \) within the corresponding field limits is the same as for the mixed type of metamagnetic transition (Fig. 2.32). However, equilibrium domain sizes of the surface metadomains are much larger than for the bulk metadomains and exist only in a much narrower field region. In particular, the critical width of isolated stripe domains of the minority phase by surface metamagnetic transition is \( d_c = 2.98 \) \( \mu m \). This value is more than twenty times larger than the corresponding value of \( d_c = 127.45 \) \( nm \) for bulk metamagnetic domains and more than six hundred times larger than the total thicknesses of the ferromagnetic blocks with \( X = 5 \). The equilibrium domain patterns can hardly be formed in such system. The energy gain in the demagnetized state becomes very small for such thin multilayers, and any trace of wall coercivity will suppress the formation of equilibrium domains [2]. That is why the surface transition in the experimental data Fig. 2.28 is characterized by strongly hysteretic loops compared to the bulk transition.

So far, only homogeneously magnetized blocks or metamagnetic (parallel) stripes have been considered as possible domain configuration. In real multilayers near the transition fields, the regular stripes transform into a system of isolated minority stripes. From the magnetization process in purely ferromagnetic multilayers with perpendicular anisotropy it is known that, close to saturation, the minority domains degenerate into bubbles, which finally collapse. Owing to the close physical relation between the metamagnetic domains and those in ferromagnetic multilayers we can give a more detailed description of the metamagnetic domain evolution in a field by including bubble domain formation, as in Refs. [20] and [II]. Thus, the instability of the stripe structure indicates the transformation of minority stripes into isolated bubbles (Fig. 2.34, Insets A, B) Isolated stripes can exist as metastable entities within broad ranges of the magnetic fields (dashed areas in Fig. 2.34). Metamagnetic bubble domains can
Figure 2.33: (Color) The equilibrium values of domain sizes as functions of the applied field in the multilayer with $N=10$, $X=5$ at bulk metamagnetic transition. The inset shows the equilibrium domain sizes at the surface metamagnetic transition for the same sample. The open circles denote the critical width of isolated stripe domains of minority phase $d_c$ at critical fields $H_{s1}$ and $H_{s2}$ (see Fig. 2.32).

exist as isolated entities or they may condense into regular hexagonal lattices. Thus, these domains can also be formed during the metamagnetic transitions. Micromagnetic equations for such domains can be readily derived from the corresponding equations for bubbles in ferromagnetic multilayers [II]. The calculated bubble collapse fields $H_{c1,2}$ and bubble strip-out (elliptic instability) fields $H_{s1,2}$ are marked in Fig. 2.34. It should be noted that metamagnetic bubble domains in AF coupled multilayers have peculiar property. In particular, regular lattices of metamagnetic bubble domains could be stabilized at zero external field, just by tuning the material and/or geometrical parameters of multilayers. In contrast, bubble domain lattices usually cannot be a favored state at zero field in single magnetic layers or in ferromagnetically coupled multilayers [2].

The MFM images, from Ref. [VII], shown in Fig. 2.35 illustrate the domain evolution with magnetic field starting from the AF state (Fig. 2.35 a). Applying small fields, the initial AF state is preserved which corresponds to the plateau region of the hysteresis (Fig. 2.29). At 0.2 T the AF state is overcome. Then ferromagnetic domains with magnetization pointing along the applied field (dark contrast) are formed in those layers with magnetization antiparallel to the field (Fig. 2.35 b). This corresponds to the simultaneous formation of surface and bulk metadomains (IV), as displayed in Fig. 2.31 for $X=8$. Increasing the magnetic field further, the up-domains with widths $d^{(+)}$ in the metamagnetic stripes grow at the cost of the down-domains with width $d^{(-)}$. This process, occurs first by increase in length of finite domain strips while their width stays essentially constant (not shown here). At an applied magnetic bias field of about 0.25 T (Fig. 2.35 c) the metamagnetic up- and down domains adopt an almost balanced configuration with a domain period $D \approx 500$ nm, as compared with the expected value of 353 nm. The theoretical estimate is in reasonable agreement
Figure 2.34: (Color) Details of the phase diagram (Fig. 2.31) for $X = 8$. Shaded (red) area indicates the region with metamagnetic domains. At critical fields $H_{s1}$ ($H_{s2}$) the metamagnetic striped phase transforms into a system of isolated ferromagnetic stripes within the saturated ferromagnetic (antiferromagnetic) state shown in Inset A (B). They can exist as metastable structures within dashed areas. Arrows indicate the critical fields for isolated bubbles. A set of point $a$−$b$−$c$−$d$−$e$−$f$−$g$−$h$−$i$−$j$ show a succession of MFM images as presented in Fig. 2.35.

with observations. Still, the deviation between the experiment and the theoretical result for the domain period is significant. The discrepancy may be explained by (i) roughness effects and hysteretic behaviour of the system (in the same way as pinning will lead to hysteresis in a field cycle, it may trap domain walls in non-equilibrium position, thereby avoiding the adjustment of equilibrium domain width), and (ii) the fact that we use average values for material parameters, such as $J$, $M_s$ and $l_c$. These parameters fit the system in general but could be slightly different in different samples and may result in some deviations of the estimates for domain sizes. Increasing the field further, the minority domains reduce in length (Fig. 2.35 d) and finally transform into isolated stripes and bubbles (Fig. 2.35 e) at around 0.4 T in very good agreement with the critical field $H_{s2}$ plotted in Fig. 2.34. At an applied field of 0.45 T all minority domains have vanished and the sample is fully saturated. Reducing the field leaves the sample in the saturated state at 0.4 T (Fig. 2.35 f), which is another manifestation of the hysteresis. Bubble nucleation and strip-out occurs between 0.40 and 0.35 T (Fig. 2.35 g) and upon decreasing the field further the bright domains (AF state) increase in length and width until the up domain become isolated and shrink (Fig. 2.35 h and i). At lowest field, these isolated minority domains vanish almost completely, leaving the multilayer in the original homogeneous AF state. In the MFM observations in Fig. 2.35 the surface and bulk metamagnetic transition remain coupled in accordance with the phase diagram in Fig. 2.31. In fact, the surface metamagnetic transition region in the case of $N = 10$ as well as for $N = 18$ is always very narrow compared to the bulk transition. Moreover, the estimated stripe domain period in the case of surface transition is extremely large. Thus, in real multilayers the nucleation of such multidomain states is hindered and the system exhibits a hysteretic behaviour with a square magnetization loop. This explains the data for $N = 10$ and $X = 5$ in Fig. 2.28, where the magnetization curve shows a pronounced hysteretic behaviour between about 0.1 and 0.2 T, and a second well separated hysteretic reversal above
The results described in this section can be summarised as following. The complex evolution of the specific (metamagnetic) multidomain states induced by external fields elucidates reorientation effects and the formation of isolated stripe and bubble domains within the saturated states and antiferromagnetic remanent state [VIII, IX]. Direct observation of metamagnetic domains confirms the theoretical description of this evolution. Within the micromagnetic approach introduced in Ref. [II] metamagnetic domains can be described by a modified model of ferromagnetic domains, Eq. (2.70) and Fig. 2.25. This allows to derive the equilibrium parameters of metamagnetic stripe and bubble domains and calculate magnetic phase diagrams (Figs. 2.27, 2.30 and 2.31). These diagrams for the field-driven equilibrium states can also provide the basis for future investigations of the hysteretic processes induced by coercivity and the dynamics at the various magnetic phase transitions in these multilayer systems.
2.4 Topological Defects and Nonequilibrium States in Antiferromagnetically Coupled Multilayers

Antiferromagnetically coupled multilayers with strong perpendicular magnetic anisotropy characterized by specific multidomain structures \[7,19,20,75,80,104,106,107\]. These spatially inhomogeneous magnetic textures can be separated into two fundamentally different groups. Regular multidomain configurations, which correspond to the global or local minima of the systems and have been discussed in detail in Sects. 2.2 and 2.3, and irregular networks of isolated domain walls and bands. The latter are inclusions of the ferro- and ferrimagnetic states trapped within the antiferromagnetic ground state. Similar to domain walls in bulk antiferromagnets they can not decay or be removed by continuous deformations of the magnetization structure. These topological defects display a large variability and their hysteretic formation strongly depends on the magnetic and temperature pre-history \[7,19,20,75,80,106,107\].

In this section we discuss the field-driven evolution of topological defects within the phenomenological theory of magnetic domains \[2\]. Our results provide a consistent explanation for the formation of specific remanent states in antiferromagnetically coupled multilayers and explains the physical mechanisms for the configurational hysteresis of multidomain states, as recently observed in experiments on \[\text{[Co/Pt]/Ru}\,20,104,106,107\] and \[\text{[Co/Pt]/NiO}\,19,75\] multilayers.

2.4.1 Ferrobands versus sharp domain walls

According to the experimental observations and theoretical analysis, the antiferromagnetic multilayers with strong perpendicular anisotropy may have antiferromagnetic single-domain structure as zero-field ground-state in certain ranges of geometry and materials parameters, see Ref. \[20\] and [III]. Planar defects separating antiferromagnetic domains in remanent states of these multilayers may arise either as sharp domain walls or as ferrobands (“shifted antiferromagnetic walls”) (Fig. 2.36 Insets (A) and (B)). Sharp walls are similar to 180° domain walls in bulk antiferromagnets. Ferrobands arise in antiferromagnetically coupled multilayers due to a subtle interplay between magnetodipole and interlayer exchange interactions. To investigate this phenomenon we consider an isolated ferroband of width \(a\) in a multilayer consisting of \(N\) identical magnetic layers of thickness \(t\) separated by nonmagnetic spacers of thickness \(S\) (Fig. 2.36, Inset (B)). The magnetic energy of this system (per unity band length) can be written in the following form [VIII]

\[
E = 4\pi M^2 t^2 N \left[ F(u) + \eta u \right], \tag{2.77}
\]

where \(u = a/t\) is the reduced band width. The magnetostatic energy \(F(u)\) is derived by solving the corresponding magnetostatic problem for a “charged” band [VIII]

\[
F(u) = \frac{1}{4\pi} \sum_{k=1}^{N-1} \left( 1 - \frac{k}{N} \right) \Xi(u, \tau k) \tag{2.78}
\]
where \( \tau = 1 + S/t \), and
\[
\Xi (u, \tau k) = 2f(u, \tau k) - f(u, \tau k + 1) - f(u, \tau k - 1),
\]
\[
f(u, \omega) = (\omega^2 - u^2) \ln(\omega^2 + u^2) - \omega^2 \ln(\omega^2) - 4\omega u \arctan(u/\omega).
\]

Here we introduce an effective magnetic coupling parameter
\[
\eta = \left( 1 - \frac{1}{N} \right) \frac{\delta}{t} - \frac{H}{4\pi M_s}.
\]

\( H \) is an applied magnetic field perpendicular to the multilayer. The exchange length \( \delta \) is given by the ratio of the antiferromagnetic coupling \( J > 0 \) and the stray-field energy, \( \delta = J/(2\pi M_s^2) \). Note that \( \eta \) includes all material parameters of the systems, while the reduced magnetostatic energy \( F(u) \) depends only on geometrical parameters of the multilayer, namely, the ratio \( S/t \).

The condition \( dE/du = 0 \) yields the equation for equilibrium ferroband widths:
\[
\eta = G(u) \equiv \frac{1}{8\pi} \sum_{k=1, \text{odd}}^{N-1} \left( 1 - \frac{k}{N} \right) \Xi_g (u, \tau k),
\]
where

\[
\Xi_g = 2g(u, \tau k) - g(u, \tau k + 1) - g(u, \tau k - 1),
\]

(2.82)

\[
g(u, \omega) = 2\left[u \ln (\omega^2 + u^2) + u + 2\omega \arctan\left(\frac{u}{\omega}\right)\right].
\]

Typical solutions of Eq. (2.81) are plotted in Fig. 2.36 for thickness ratios \(S/t\) corresponding to geometrical parameters in different experimentally investigated systems: \(S/t = 0.36\) [106], 0.19 [20] and 0.06 [116]. Note that a sharp domain wall can be treated as the limiting case of a ferroband with zero width.

The stability condition, \(d^2E/du^2 = 0\), gives the equation

\[
\sum_{k=1 \atop k \text{odd}}^{N-1} \left(1 - \frac{k}{N}\right) \ln \left[1 + \frac{1 + 2u^2 - 2\tau^2 k^2}{(\tau^2 k^2 + u^2)^2}\right] = 0
\]

(2.83)

which, combined with Eq. (2.81), determines critical values of the ferroband width \(u_c\) and \(\eta_c\). Because Eq. (2.83) does not include the material parameters, the solutions for \(u_c\) are functions of the ratio \(S/t\) alone. In particular, for bilayers

\[
a_c = \sqrt{S^2 + 2tS + t^2/2}.
\]

(2.84)

By substituting \(u_c\) into Eq. (2.81) we find \(\eta_c = G_c \equiv G(u_c)\). The analysis shows that solutions of Eq. (2.81) exist in the range \(0 < \eta < \eta_c(S/t)\). The equations \(\eta = 0\) and \(\eta = \eta_c(S/t)\) yield the limiting fields where ferroband solutions can exist as stable defects

\[
H_1(t) = 4\pi M \left(1 - \frac{1}{N}\right) \frac{\delta}{t},
\]

\[
H_2(t) = H_1(t) - G_c(S/t).
\]

(2.85)

In the magnetic phase diagram critical fields \(H_1(t)\) and \(H_2(t)\) confine the region with metastable ferroband defects.

For a bilayer the magnetic phase diagram (Fig. 2.37) includes the following equilibrium states: the single domain antiferromagnetic (below \(g - c - b\)) and ferromagnetic (above \(g - d - e\)) phases, the ferrostripe (below e-d-c-b) and metamagnetic \((g - c - d - g)\) phases. In the metamagnetic phase only one of the ferromagnetic blocks includes multidomain states while another layer is homogeneously saturated along the applied field [20] and [VIII]. Inset (C) in Fig. 2.36 shows the evolution of the energy profiles \(E(a)\) when the bias field varies from \(H = H_1\) (profile 1) to \(H = H_2\) (profile 4). At the critical field \(H_1\) (line \(g - c'\) in Fig. 2.37) the energy of the antiferromagnetic phase equals the energy of the ferromagnetic (saturated) state. Here, both sharp domain walls \((a = 0)\) solutions and ferrobands expand to infinity (profile 1). For lower fields both topological defects are locally stable (profile 2). In decreasing bias field the ferroband energy gradually increases. At a certain value of the bias field \(H^*(t)\) the energies of both defect types become equal (profile 3), and for \(H^*(t) > H > H_2(t)\) the ferroband energy is larger than that of the sharp domain wall. Finally at the critical field \(H_2(t)\) the ferrobands collapse (profile 4), and for \(H < H_2(t)\) only sharp wall solutions can exist (profile 5). Because the variation of the ferroband width does not change the domain wall
Figure 2.37: (Color) Figure (a) is the details of phase diagram of ground states for the multilayer with $N = 2$, $X = 1$ and fixed thickness of interlayer $S/l_c = 0.1$ about the transition region between the homogeneous antiferromagnetic phase and the ferrostripes. The first-order transition line between these states is shown by a thick (blue) line $a - b - b'$. The ferrostripes are metastable between $o - k$ and $o - b - b'$. The ferromagnetic bands (Fig. 2.36, Inset B) exist within the shaded area $o - a - a'$ and $o - b - a'$ lines. Thick (red) line $\delta/l_c = 0.1$ corresponds to the horizontal axis ($H=0$) in figure (b), $t_a = 0.401$, $t_b = 0.923$. Figure (b) is the magnetic phase diagram of the equilibrium states in reduced variables for layer thickness $t/l_c$ and bias field $H/(4\pi M_s)$ (see also Fig. 2.26). The hatched area as well as in figure (a) shows the existence region of ferrobands. The lower panel indicates thickness intervals for the different types of remanent states. At the critical line $H_1$ ($g - c'$) the ferroband transform into saturated (ferromagnetic) states by an unlimited expansion of their sizes ($a \to \infty$). At the line $H_2$ ($f - a$) the ferrobands transform into sharp domain walls. The $b - c - c'$ line marks the transition into the multidomain ferrostripe phase. The inset shows the ferroband width as a function of an opposing (negative) magnetic field (solid line) for a [Co/Pt]NiO bilayer investigated in Ref. [116]. (Line for present theory, points experimental data [116]).
energy the equilibrium ferroband sizes do not depend on the characteristic length. They are formed only under competing influence of the antiferromagnetic exchange and the combined external bias and dipolar stray fields.

2.4.2 Reorientation effects and remanent states

Topological defects can not arise spontaneously. However, they can be induced by magnetization processes. Thus, the formation and evolution of topological defects strongly depend on the sequence of magnetic-field-driven states and the transitions between them. Usually antiferromagnetic domain walls and ferrobands arise in the remanent state after demagnetization [20,106]. This follows from the fact that antiferromagnetic domain walls and ferrobands are remnants of the ferromagnetic phases within the antiferromagnetic matrix. These wall defects also arise after in-plane demagnetization however, the defected antiferromagnetic state, i.e. the domain pattern may own different sizes and morphologies after different field histories [20]. But, the structure of the topological wall defects should be the same in either case. Depending on the magnetic layer thickness remanent states consist of multidomain patterns with sharp domain walls \( t < t_a \) or ferrobands \( t_a < t < t_b \). In order to avoid any possible misunderstanding we note that these are defect patterns of the spatially homogeneous antiferromagnetic state owing to topologically stable distortions in the form of irregular domain walls or ferrobands. For \( t > t_b \) the remanent states consist of the regular ferrostripe phase (Fig. 2.37). In multilayers with thicker magnetic layers the antiferromagnetic and ferromagnetic phases are separated by the region of transitional domain structures \( (\text{metamagnetic domains}) \) (Fig. 2.37). In this case the antiferromagnetic phase may include remnants of metamagnetic domains. Such textures have been observed in \([\text{Co/Pt}]\text{Ru multilayers} \) after out-of-plane saturation [20]. Experimental data on the variation of the ferroband size under influence of the applied field have been reported in Ref. [116]. In the experimental investigation a ferroband in a \([\text{Pt(5Å)/Co(4Å)]_4}/\text{NiO(11Å)/[Co(4Å)/Pt(5Å)]_4} \) bilayer was squeezed.
by an opposing magnetic field. By fitting the experimental data of Ref. [116] we calculate
from Eq. (2.81) the exchange constant $J = 0.002 \text{ erg/cm}^2$ ($\delta = 0.063 \text{ nm}$) and the optimal
ferroband width as a function of the bias field, $a(H)$ (Fig. 2.37 b, Inset). This value of $J$ is
in reasonable agreement with those for [Co/Pt]NiO multilayers investigated in Ref. [19].

According to experimental observations ferrobands can exist either in a single domain
state [19, 106, 116] or split into a system of domains creating, so called “tiger-tail” pat-
terns [20, 104]. The “tiger-tail” multidomain states of these defects clearly are due to de-
 polarization. In principle, these effects can be considered by additional stray field terms
for the modulated magnetization along a ferroband in the model energy (2.77). Further ex-
perimental investigations of “tiger-tail” patterns together with a micromagnetic analysis of
these multidomain patterns should give deeper insight into the formation and evolution of
topological defects in this class of magnetic nanostructures.

In conclusion, we present an exhaustive analysis of specific topological defects (ferrobands)
arising in perpendicular antiferromagnetically coupled multilayers. Our analytical solutions
generalize and complete numerical studies of these defects in Refs. [19, 20, 104, 107]. Magnetic-
field-driven evolution and transformation of ferrobands explain the formation of defected re-
manent states recently observed in [Co/Pt]Ru and [Co/Pt]NiO antiferromagnetic multilayers.
3 Application of Stripe Domain Theory for Magnetic Force Microscopy on Multilayers with Interlayer Exchange Coupling

The remarkable role of stray field effects in synthetic antiferromagnets and the peculiarities of their multidomain states are currently investigated by high resolution magnetic force microscopy (MFM) (for recent examples of experimental tests on domain theory by MFM see, e.g., Refs. [7, 117]). From the theoretical side, only few results have been obtained on MFM images in antiferromagnetically coupled multilayers, mostly by numerical methods [6,19,118]. Here we present an analytical approach that provides a comprehensive description of stray field distributions and MFM images from multidomain states of these nanostructures. This chapter is based on results of Ref. [VI]. We show that the stray field components and their spatial derivatives, that are crucial for an analysis of MFM contrast, own distinctive features for different multidomain states. These features allow to recognize the particular distribution of the magnetization at the surfaces of domains and in the depth of the multilayers. The quantitative relations from theory for the MFM contrast can also serve to determine the values of magnetic interactions, i.e. materials parameters of an antiferromagnetic multilayer. We apply our results for an analysis of multidomain states observed in [Co/Pt]Ru multilayers [7].

3.1 Introduction to Magnetic Force Microscopy

The magnetic force microscope is a variant of the scanning (or “atomic”) force microscope [120]. It records the magnetostatic forces or force gradients between a sample and a small ferromagnetic tip. The two most prominent advantages of the technique contributing to its success are its potential insensitivity to non-magnetic surface coatings and relief, and good resolution down into the nanometre range [121]. Reviews of the principles and methods of MFM can be found in [122,123].

In force microscopy, forces are measured by the deflection of a flexible beam, the cantilever, which carries the tip-shaped probe at its free end. It can be adjusted by a piezoelectric actuator and its position usually is detected by optical transducers. The control signal can be used to operate the scanning microscope in different modes. One option is to run it at constant force (equivalent to a constant deflection of the cantilever) and use the height necessary to obtain this state as the imaging information. Another mode consists in operating the elastic tongue at a frequency close to its mechanical resonance, and to detect any change in the resonance amplitude or shift in phase. Since a magnetic force gradient is equivalent to an additional contribution to the spring constant of the cantilever, profiles of constant
force gradient can be recorded in this way. The concept of magnetic force microscope is schematically illustrated in Fig. 3.1.

Phase detection and frequency modulation give the best results, with a higher signal-to-noise ratio. The signal depends on the force derivative in the following manner:

\[
\Delta \Phi = \frac{\Theta}{\kappa} \left( \frac{\partial F_z}{\partial z} \right), \tag{3.1}
\]

where \(\Theta\) is the quality factor and \(\kappa\) is the cantilever spring constant.

An attractive interaction \((\partial F_z/\partial z > 0)\) leads to a negative frequency shift, while a repulsive interaction \((\partial F_z/\partial z < 0)\) gives a positive frequency shift. The force derivative can originate from a wide range of sources, including electrostatic tip-sample interactions, van der Waals forces, damping, or capillary forces. However, MFM relies mainly on those forces that arise from a long-range magnetostatic coupling between tip and sample. This coupling depends on the internal magnetic structure of the tip, which greatly complicates the mechanism of contrast formation. However, at short distances, it is difficult to separate the magnetic interactions from van der Waals interactions. Taking advantage of the fact that topographic interactions are short range while magnetic interactions are long range, one uses the lift-mode technique: take an image of the sample at short distances to obtain primarily topographic information, then use this information to keep the tip at a fixed height \(z_0\) above the sample, following the topography, and thereby obtain an almost purely magnetic image.

Note that with respect to speed, magnetic force microscopy cannot compete with optical or electron microscopy techniques. This is no disadvantage in the investigation of information storage patterns which should be quite stable in time. In regular domain observation this limitation also is acceptable.

### 3.2 Stray Field of Stripe Domains

We consider strong stripes, i.e. band domains in a superlattice composed of \(N\) identical layers of thickness \(T\) separated by spacers of thickness \(S\), (see, Fig. 2.20). The stripes with alternating magnetization \(\mathbf{M}\) along the \(z\) direction and with \(|\mathbf{M}| = M \equiv \text{const}\) have the period length \(D\) and are separated by domain walls of vanishingly small thickness. As a result of
competition between interlayer exchange coupling and dipolar interaction and domain wall energy three different ground states can be realized depending on the materials and geometrical parameters of the multilayer [III, VII] (see Sect. 2.2). The phase diagrams considering these stripe ground states show that both types of stripes can exist as stable or metastable states in extended and overlapping ranges of the material parameters [III]. The extended coexistence regions of different types of multidomain states in the phase diagrams also entails the possibility to create complex “interspersed” patterns that consist of subdomains with ferro and antiferro stripes. For identical values of the materials parameters the equilibrium domain widths for ferro and antiferro stripes can differ considerably (see Fig. 2.24). Hence, the mixed stripe patterns can include regions with different domain sizes. Similar structures have been observed in some [Co/Pt]Ru multilayers [20]. In addition to the differing characteristic periods of ferro and antiferro stripes, these stripe patterns also cause different distributions of the stray fields $H^{(m)}$ at the sample surfaces. The stray fields can be probed by magnetic force microscopy, however, the properties of the stray fields peculiar to the different stripe patterns are rather subtle. Thus, the experimental observation and quantitative evaluation of these differences must be based on a detailed comparison with theoretical model calculations.

The stray field above the sample surface can be expressed as a superposition of the stray fields from the $2N$ interface planes with “charged” stripes (Inset in Fig. 3.2). By solving the magnetostatic problem for a plane with “charged” stripes one can write the solutions for the stray field components $h^{(m)}(x, z)$. In particular, the scalar potential of a sheet with “charged” stripes in zero external field can be derived by solving Poisson’s equation (see Sect. 1.7.1 Eq. (1.44))

$$\phi(x, z) = \frac{4MD}{\pi} \sum_{n=1}^{\infty} \frac{\sin(\pi n/2)}{n^2} \cos \left(\frac{2\pi nx}{D}\right) \exp \left(-\frac{2\pi nz}{D}\right). \quad (3.2)$$

Using the identity $n^{-m} = [(m - 1)!]^{-1} \int_0^\infty t^{m-1} \exp(-nt) \, dt$ the Eq. (3.2) can be transformed
into the following form (see Ref. [92] and [III])

\[
\phi(x, z) = \frac{8MD}{\pi} \int_0^\infty \frac{\cos(2\pi x/D)}{\cosh^2(t + 2\pi z/D) - \sin^2(2\pi x/D)} \, dt.
\] (3.3)

From this closed expression, the components of the stray field \( h^{(m)} = -\nabla \phi \) are readily derived in the analytical form:

\[
h^{(m)}_x(x, z) = 2M \ln \left| \frac{\cosh(2\pi z/D) - \sin(2\pi x/D)}{\cosh(2\pi z/D) + \sin(2\pi x/D)} \right| ,
\] (3.4)

\[
h^{(m)}_z(x, z) = 4M \arctan \left( \cos(2\pi x/D) / \sinh(2\pi z/D) \right),
\] (3.5)

Then the total stray field of the multilayer at a distance \( z_0 \) above the surface can be written as

\[
H^{(m)}(x, z_0) = \sum_{k=0}^{N-1} \Gamma_k \left[ h^{(m)}(x, z_0 + Tk) - h^{(m)}(x, z_0 + Tk + h) \right].
\] (3.6)

The factor \( \Gamma_k = (-1)^k \) holds for antiferro stripes, and \( \Gamma_k = 1 \) for ferro stripes. For the shifted ferrostripes the stray field has a similar form

\[
H^{(m)}(x, z_0) = \sum_{k=0}^{N-1} \left[ h^{(m)}(x + a \cdot \Gamma_k, z_0 + Tk) - h^{(m)}(x + a \cdot \Gamma_k, z_0 + Tk + h) \right],
\] (3.7)

where \( \Gamma_k = [1 - (-1)^k]/2 \).

Peculiarities of the stray-field profiles \( H^{(m)}_z \) imposed by the exchange shift, as seen in Fig. 3.2, can be measurable by magnetic force microscopy imaging. However, as it was mentioned in Sect. 2.2 the value of this shift usually is very small (\( a \approx 5-20 \text{ nm for } D \approx 260 \text{ nm, see Sect. 2.2} \)). In this case the weak peculiarities of MFM signal can be observed at very small distance from the surface only (see Fig. 3.2). The typical distance between MFM tip and sample surface used in experiment is about 20-50 nm. In this range of the distance, as seen from Fig. 3.2, the peculiarities of the stray field are hardly distinguishable. On the background of the unavoidable noise they can not qualitatively and quantitatively reveal the existence of lateral shifts. Therefore, we can neglect this small shift for the analysis of MFM images with present instruments. Below we consider the ferromagnetic stripes with zero shift only.

Spatial derivatives of \( H^{(m)}_z \) with respect to \( z \) are important for the analysis of the MFM images. The derivative can be calculated analytically by differentiation of Eq. (3.6) as

\[
\Upsilon_n(x, z_0) = \frac{\partial^n}{\partial z^n} H^{(m)}_z = 4M \left( \frac{2\pi}{D} \right)^n \sum_{k=0}^{N-1} \Gamma_k [v_n(x, z_0 + Tk) - v_n(x, z_0 + Tk + h)].
\] (3.8)

Here, we introduce a set of functions \( v_n(x, z) \) which are derivatives of the function \( v_0(x, z), \)
as defined in Eq. (3.5), with respect to the normalized geometry parameter $\xi = 2\pi z/D$,

$$v_n(x, z) \equiv \partial^n v_0 / \partial \xi^n = \cos(2\pi x/D) \frac{G_n(x, z)}{g_n^+(x, z)},$$

(3.9)

where

$$g_\pm = [\cosh(4\pi z/D) \pm \cos(4\pi x/D)]/2,$$

$$G_1 = -\cosh(2\pi z/D),$$

$$G_2 = \sinh(2\pi z/D)(1 + g_-),$$

$$G_3 = -\cosh(2\pi z/D)(2g_-^2 - g_+^2 + 2g_+ - 2),$$

$$G_4 = \sinh(2\pi z/D)\left[6(g_- + 1)^2(g_- - 1) + g_+^2(1 - 5g_-)\right].$$

(3.10)

Combined with the equation $dw_N/dD = 0$, which determines the equilibrium domain period, Eqs. (3.6) and (3.8) describe the stray field and its spatial derivatives as a function of the coordinates $x, z$ for a multilayer in a stripe state. The stray field $H^{(m)}(x, z)$ (3.6) and the derivatives $\Upsilon_n(x, z_0) (3.8)$ are expressed as sets of analytical functions in Eqs. (3.4), (3.5) (3.9), and (3.10). The expressions depend on the geometrical parameters and, via the equilibrium domain widths $D$, on the material parameters of the multilayer. These analytical expressions can be readily evaluated by elementary mathematical means.

### 3.3 Applications

In order to demonstrate the main features of the stray fields $H^{(m)}_z(x, z)$ from the ferro and antiferro stripes we evaluate these functions for a multilayer [[Co/Pt]$_7$CoRu]$_4$ with magnetic and geometrical parameters corresponding to a sample that was investigated experimentally in Ref. [7]. In this superlattice the ferromagnetic constituents are magnetic [Co/Pt]$_7$ multilayers with thicknesses of the ferromagnetic Co-layers 0.4 nm and thickness of the Pt layer 0.7 nm. The non-ferromagnetic Ru spacer has a thickness of 0.9 nm and mediates an indirect antiferromagnetic interlayer exchange. The domain period has been determined as $D = 260$ nm [7]. For ferro and antiferro stripe modes the functions $H^{(m)}_z(x, z)$ are markedly different both in the intensity and in the location of characteristic extremal points (Fig. 3.3). Moreover they display qualitatively different functional dependencies on the distance from the multilayer surface $z_0$ (see Insets in Fig. 3.3).

The stray field distribution over the multilayer surface can be viewed as a superposition of magnetostatic fields from systems of “charged” bands. This allows to give a simple physical interpretation for the main features of the stray field profiles in Fig. 3.3. First we consider ferro stripes. It is convenient to separate the total stray field over a domain into two contributions: one created by the top and bottom bands of the domain itself (this we may call a “self” field of this single domain), and the stray field contributions produced by all other “charged” bands. Above the domain centers near the sample surface, $z_0 \ll D$, the stray “self” field is small due to the screening effects of the domain top and bottom surfaces. Because the bands change their “charge” at the domain walls the stray field is substantially enhanced above the walls. As a result, the profiles $H^{(m)}_z(x, z)$ have characteristic wells in the domain centers for $z_0 \ll D$. For increasing distance $z_0$ from the surface the difference between values of $H^{(m)}_z$ above the
Figure 3.3: (Color) Calculated stray field profiles $H_z^m(x)$ for ferromagnetic a) and antiferromagnetic b) modes for $[[\text{Co/Pt}]_7\text{CoRu}]_4$ multilayers investigated in Ref. [7]. Insets show the perpendicular stray field component $H_z^m(z_0)$ at the center of the stripes in dependency on the distance $z_0$ above the multilayer surface. In ferro stripes this function monotonically decreases with increasing $z$, while in antiferro stripes it has a maximum at a finite distance from the surface.

center and at the domain edges decreases due to the increasing influence of neighbouring poles. For large distances $z_0$ the wells disappear and the profiles obtain a typical bell-like shape (compare the traces for $z_0 = 10$ and 30 nm in Fig. 3.3 (a)). The antiferro stripe mode can be obtained from those for ferro stripes by changing the magnetostatic “charges” for the bands in all even layers. This weakens the total stray field and sharpens the difference between the stray fields at the center and near the domain edges. Finally, the competing character of the stray field contributions from odd and even layers causes the nonmonotonic dependence of $H_z^m(z)$ (Inset in Fig. 3.3 (b)).

Generally, the functions $H^m(x, z_0)$, $\Upsilon_n^F(x, z_0)$, $\Upsilon_n^{AF}(x, z_0)$ in Eqs. (3.6) and (3.8) have a number of characteristic features that can be utilized in new methods to investigate the multidomain modes. One method can be based on measuring the MFM contrast in the center of the domains. In this case $x = kD$, $k = 0, 1, 2, 3...$ and the functions $\tilde{\upsilon}(z) \equiv \upsilon_n(kD, z)$ are reduced to the following expressions

$$\tilde{\upsilon}_0(z) = \arccos \left[ \tanh \left( 2\pi z / D \right) \right],$$

$$\tilde{\upsilon}_1(z) = - \cosh^{-1} \left( 2\pi z / D \right),$$

$$\tilde{\upsilon}_2(z) = \sinh \left( 2\pi z / D \right) \cosh^{-2} \left( 2\pi z / D \right),$$

$$\tilde{\upsilon}_3(z) = -(1 - \sinh^2 \left( 2\pi z / D \right)) \cosh^{-3} \left( 2\pi z / D \right),$$

$$\tilde{\upsilon}_4(z) = \sinh \left( 2\pi z / D \right) \left( \sinh^2 \left( 2\pi z / D \right) - 5 \right) \cosh^{-4} \left( 2\pi z / D \right).$$

(3.11)

Profiles $\tilde{\upsilon}_n(z_0)$ from Eqs. (3.11) for $n = 1, 2, 3, 4$ are plotted in Fig. 3.4. Their characteristic features can be used to ascertain the type of the stripe mode and should even allow quantitative evaluation of magnetic properties in multilayers, if and when they are accessible in experiment.

Owing to the properties of the magnetic probe, a signal from a magnetic force microscope
Figure 3.4: (Color) Characteristic functions $\tilde{v}_n(z_0) \equiv v_n(0, z_0)$ (Eq. (3.11)) describe MFM images in the center of domains. Inset indicates the location of the extrema of the functions $v_n(x, z_0)$ in the $xOz$ plane.

generally differs strongly from the $H_z(x)$ profile. In order to compare the expected outcome of different domain configurations, the MFM contrast has to be calculated for realistic tip models. The MFM signal for a magnetic cantilever oscillating in $z$-direction is given by [124, 125]

$$\Delta \Phi = -\frac{\Theta}{\kappa} \left( \frac{\partial F_z}{\partial z} \right) = -\frac{\Theta}{\kappa} \mu_0 \int_{\text{tip}} \frac{\partial^2 \left[ M^{(\text{tip})}(r) \cdot H^{(m)}(r) \right]}{\partial z^2} dV. \quad (3.12)$$

Here, $\Delta \Phi$ is the measured phase shift between excitation and oscillation due to the force gradient $\partial F_z/\partial z$ that acts on the cantilever in the stray field of the sample $H^{(m)}(r)$. Assuming a rigid tip magnetized in $z$-direction, i.e., a tip with a homogeneous magnetization distribution, $M^{(\text{tip})} \equiv \text{const}$ throughout the tip volume, that does not change during the scan across the stray field of the sample, the expression for the force gradient simplifies to

$$\frac{\partial F_z}{\partial z} = M_z^{(\text{tip})} \int_{\text{tip}} \frac{\partial^2 H_z^{(m)}(r)}{\partial z^2} dV. \quad (3.13)$$

The volume integration is a crucial step as it modifies the signal compared to the profile estimated by the second stray field derivative.

A realistic tip geometry can be modelled by a truncated triangle placed in the $x-z$-plane (see inset in Fig. 3.7). This mimics the two parallel sides of the 4-sided pyramidal geometry of a typical MFM tip. The two-dimensional model considerably simplifies the calculations. The error incurred by this reduced model is minor because of the infinite extension of the domain models in the $y$-direction.

To demonstrate this approach we analyze the MFM contrast measured across an available thick [(Co/Pt)$_8$CoRu]$_{18}$ multilayer prepared at Hitachi GST (for details on these multilayers, see Refs. [7]). The MFM pictures show a typical maze pattern of ferro stripes with
Figure 3.5: (Color) a) Maze domain pattern in \([[\text{Co/Pt}]_8/\text{Co/Ru}]_{18}\) observed by MFM at room temperature. b) Line scans along the marked line for \(z_0 = 10, 20, 30, 40, 50,\) and 60 nm (decreasing contrast amplitude for increasing \(z_0\)).

perpendicular magnetization at room temperature Fig. 3.5 (a)). For comparison with contrast calculations the MFM signal along the marked line of the image was recorded repeatedly (with the slow scan axis disabled) at lift heights of 10, 20, 30, 40, 50, and 60 nm. Averaged scan lines for each lift height in Fig. 3.5 (b) display a reduced contrast for increasing scan height. The very regular domain pattern in the center of this scan (framed area) was modeled according to Eq. (3.8) as parallel ferro stripes with the period of 360 nm based on a \(N = 18\) multilayer with the known layer architecture. Due to the larger layer thickness, as compared to the multilayer model of Fig. 3.3, the stray field profiles \(H_z(x)\) do not possess a minimum at the domain center even for the smallest achievable scan height of 10 nm (see Fig. 3.6). Still, we may employ these test scans to check and justify the simulation procedure for MFM contrast. With the resulting second stray field derivatives the MFM phase shift was computed according to Eq. (3.8) using the above mentioned tip model.

Figure 3.6: (Color) Experimental (points) and calculated (solid lines) phase shift profiles \([[\text{Co/Pt}]_8\text{CoRu}]_{18}\) for MFM scans corresponding to the framed area in Fig. 3.5(b).
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(a) b) c)

Figure 3.7: (Color) Force gradient profiles $\partial f(x)/\partial z$ calculated with a truncated triangular tip model for a) ferro, and b) antiferro stripes of a [[Co/Pt]$_7$CoRu]$_4$ multilayer. c) Force gradient at the center of the stripes in dependency on the distance $z_0$ above the multilayer surface (geometrical parameters as in Fig. 3.3).

parameters a spring constant of $\kappa = 2$ N/m and a quality factor of $\Theta = 100$ were used. The height, pyramid angle, and tip apex were chosen as 4 $\mu$m, 30°, and 50 nm, respectively, and the film coating was assumed to be 30 nm. As an adjustable parameter the tip magnetization $M_{\text{tip}}$ was set to $3 \cdot 10^5$ A/m. With this reasonable value the computed phase shift magnitude, line profile, and lift height dependency compare very well with the experimental data (Fig. 3.6) considering the unavoidable deviation from regularity of spacing and shape for the experimental domains as compared to the idealized parallel strong ferro stripes of the model (Fig. 3.3). Such calculations can thus be used to predict differences in the MFM contrast of distinct types of stripe domains.

Fig. 3.7 shows the calculated force gradient profiles for the ferro and antiferro stripes presented in Fig. 3.3. The profiles reflect general features of the stray field distributions in the multidomain patterns. Here as well, quantitative and qualitative differences allow to distinguish ferro and antiferro stripes. The signal from the antiferro stripes is clearly weakened, but it is large enough to be measured in a typical MFM setup which allows the detection of a few $10 \mu$N/m ($10^{-2}$ dyn/cm) [126]. However, as quantitative MFM measurements are still rare and require precise calibration routines [125, 126] the absolute value of the signal is not a reliable criterion for the distinction between different stripe configurations. More importantly, the force gradient directly above the center of a domain shows a monotonically decreasing signal strength for increasing scan height $z_0$ in case of the ferro stripes. Above the antiferro stripes, on the other hand, the force gradient is increasing with increasing scan height, at least in the range from 10 to 30 nm. This qualitative difference is demonstrated again in Fig. 3.7(c), where the force gradient experienced by a realistic tip model is plotted as a function of $z_0$ for the two cases. The non-monotonic behavior of the force gradient observed above a multidomain structure can be taken as a clear fingerprint of an antiferro stripe state within a multilayer stack. The calculation even reveals a sign change as an additional signature of the antiferro stripes, but this appears at a scan height smaller than 10 nm, which is experimentally very difficult to access, as topographic irregularities can influence the measurement.

The solutions for the stray field (3.6) and its derivatives (3.8) can be simplified in the
practically important case of large domains as compared to the thickness of the multilayer stack, $D \gg N t$. Expansion of $\Upsilon_n(x, z_0)$ with respect to the small parameter $t/D \ll 1$ yields for ferro stripes

$$\Upsilon_n^{(F)}(x, z) = -4MN (2\pi/D)^n \upsilon_{n+1}(x, z),$$

(3.14)

and for antiferro stripes with even $N$

$$\Upsilon_n^{(AF)}(x, z) = -2MN (2\pi/D)^n (1 + s/h) \upsilon_{n+2}(x, z).$$

(3.15)

For antiferro stripes in multilayers with odd $N$ the functions $\Upsilon_n(x, z)$ are given by Eq. (3.14). Note that the functions $\Upsilon_n^{(F)}$ from the Eq. (3.14) are proportional to $\upsilon_{n+1}$, while the functions $\Upsilon_n^{(AF)}$ in Eq. (3.15) can be expressed by derivative of the functions $\upsilon_{n+1}$ through the relation $\upsilon_{n+2} = \partial \upsilon_{n+1}/\partial z$. In this limit of large domains, the functions $\Upsilon_n$ in the antiferro mode behave as $z$-derivatives of the corresponding functions of ferro modes. In particular for $n = 0$, the Eqs. (3.14) and (3.15) give the perpendicular stray field components for ferro stripes and antiferro stripes with odd $N$, correspondingly, by the expressions

$$H_z^{(F)}(x, z_0) = -4MN \upsilon_1(x, z_0),$$

$$H_z^{(AF)}(x, z_0) = -2MN(1 + s/h) \upsilon_2(x, z_0).$$

(3.16)

### 3.4 Conclusions

We have presented analytical solutions for the stray field (Eq. (3.6)) and its spatial derivatives (Eq. (3.8)) in multidomain states of magnetic multilayers with out-of-plane magnetization. These solutions can be applied to calculate the MFM contrast for a realistic tip geometry using Eq. (3.13). It is shown that the ground state ferro and antiferro stripe structures in antiferromagnetically coupled multilayers differ by their period lengths and by the spatial distribution of their stray fields. Our analytical calculations executed within a simplified model of one-dimensional multidomain patterns with fixed magnetization orientation and infinitely thin domain walls can be employed to model general features of MFM images from perpendicular multilayers, e.g. for systems with or without (antiferromagnetic) interlayer exchange. These features of the multidomain pattern can be used to identify different domain states, as ferro and antiferro stripes, and to extract values of the magnetic interactions. The analytical stripe domain models also create a basis for more detailed investigations on more realistic models. Such models should consider distorted magnetization distributions, where the magnetization is tilted away from the perpendicular direction in sizeable fractions of the domains due to the finite strength of uniaxial anisotropy, or a finite width of the domain walls between domains combined with the appearance of magnetic charge distributions at the walls.

Within the scope of regular one-dimensional stripe structure we have analyzed the basic structure for spatially inhomogeneous distributions of the magnetization within the multilayer and stray fields above its surface. It was shown that ferro stripes are unstable with respect to a lateral shift of domains in adjacent layers. This creates specific multidomain patterns with distinct modulations across the multilayer stack. Additionally, in Sect. 2.4 it was shown
that in the antiferromagnetic ground state a large variety of topological defects as ferrobands, sharp domain walls and “tiger tails” can appear as a metastable state. Such effects should be consistently taken into account for the future investigation on antiferromagnetically coupled multilayers.

The calculation of the corresponding stray field distribution above such multilayer can be performed by similar methods as employed in this chapter.
4 Stripe Domains in Magnetic Shape Memory Materials

This chapter is devoted to our findings in domain structure and magnetization reversal in magnetic shape memory material. This chapter cites experimental results on direct observation of domain structure and investigation on magnetization reversal processes derived by Y.W. Lai, J. McCord, and group leader R. Schäfer in IFW Dresden with the assistance of A. Böhm from Fraunhofer IWU Dresden/Chemnitz. The collaboration with this experimental group and their results has motivated our development of a theory of stripe domains in twinned ferromagnetic single crystalline material.

This chapter is organized as follows. In the first section we give short introduction into magnetic shape memory alloys. Then, in the second section, the experimental methods and experimental observations are reported. This section is a sequel to the first experimental work reported in Ref. [127]. In section 4.3 the domain model is introduced and the results on the evolution of the variant phases with magnetic field are presented in section 4.4. Finally, we discuss the results in relation to the experimental data.

4.1 Introduction to Shape Memory Alloy

Ferromagnetic shape memory alloys as Ni-Mn-Ga, Ni-Mn-Al, or Fe-Pd constitute a class of active smart materials, which are intensively studied during past years [128, 129]. Martensitic phases of these compounds exhibit giant magnetic field induced strain transformations exceeding by orders of magnitude the best result in traditional actuator materials such as piezoelectric or magnetostrictive solids [129–131]. This remarkable phenomenon arises as an interplay of two physical effects: (i) A martensitic transition creating competing phases, i.e. crystallographic domains or variants, which are crystallographically equivalent but have different orientation. (ii) High uniaxial magnetocrystalline anisotropy that pins the magnetization vectors along certain directions of these martensite variants. Then, an applied magnetic field can drive a microstructural transformation by which the martensitic twins are redistributed. This magnetic-field drive reorientation of crystallographic variants is responsible for the huge strains and shape changes in magnetic shape memory materials. This field-driven process couples the transformation in the martensitic microstructure with conventional magnetization processes in uniaxial easy-axis ferromagnetic materials, such as displacements of domain walls and magnetization rotation within the domains, see Fig. 4.1.

The mechanism by which a magnetic field transforms the magnetic domains is illustrated schematically in Fig. 4.1. As is known, the cooling of shape memory alloys to temperatures below the martensite transformation leads to the formation of self-accommodated martensitic variants (H=0, Fig. 4.1). In this representation, magnetic domain walls that bend the magnetization vector approximately by 90 degree run along the twin-boundaries, which connect
the different martensitic variants. Under certain conditions a magnetic field $\mathbf{H}$ can lead to growth of martensitic variants whose magnetic moment is oriented favorably with respect to the magnetic field. This process changes the shape of the sample. Ideally, at a certain critical value of the magnetic field all martensitic variants align themselves along the direction of the magnetic field.

It is possible to calculate the maximum theoretical strain for the single crystals using lattice parameters of the martensite phase. If the stress applied during cooling is sufficient for single variant formation, the martensite variant with the shortest axis along the applied stress direction can be obtained instead of a self-accommodated twinned microstructure. The lattice parameters for Ni$_2$MnGa alloys in the martensite tetragonal (5M modulated) structure are $a = 0.595$ nm and $c = 0.560$ nm (compare with $a_a = 0.584$ nm for austenite, cubic phase) [132]. Austenite-martensite transformation occurs in the range of temperature from 200K for pure Ni$_2$MnGa and increases with an increase in Ni concentration up to 625K with Ni$_{2+x}$Mn$_{1-x}$Ga where $x = 0.36$. If the single variant martensite with easy axis along the strain measurement direction is set as the reference configuration, then the largest possible strain value is simply determined as

$$\frac{a - c}{c} \times 100 = 6.25\%.$$ (4.1)

This theoretical values for maximum reorientation strain, $\sim 6\%$, is in good agreement with experimental data [133].

According to recent experimental observations [127,134–141] multi-variant patterns in the archetypical and best investigated Heusler alloys Ni-Mn-Ga are formed under the combined influence of elastic interactions and demagnetization effects. The field-driven variant redistribution in the microstructures of ferromagnetic martensitic single-crystal plates takes place in a coarse microstructure, i.e. new twins nucleate as plates that extend through the whole sample plate (Fig. 4.2). Therefore, the characteristic size of the coarsest feature in the microstructure and the sample size are the same in these experiments. Starting from a macroscopic single-variant state in a single-crystalline plate, new variants with favorable magnetic anisotropy axis are nucleated on apparently preformed crystal defects. The microstructure and magnetic

Figure 4.1: (Color) Redistribution of martensitic variants in a magnetic field. Variants that are favorably oriented with respect to the applied magnetic field grow at the expense of unfavorably oriented variants [128].
domain structure cover macroscopic lengths in a macroscopic sample such as the single crystalline plates investigated in this work. The microstructures are detectable by direct light-microscopy observations. Also the magnetic domain structure in these samples can be studied by magneto-optical techniques using indicator films. [127] Consequently, the microstructural transformation processes and the micromagnetic domain structure can be visualized on the sample surface. In principle, investigations on these processes under controlled experimental conditions for simple geometries of the single-crystal sample, the applied field (and external stresses) can give information about key parameters of magnetic shape-memory materials, such as nucleation barriers for new variants, the twin-boundary energies or domain-wall energy for the 90-degree walls hitched onto the domain walls, and the twin-boundary mobility. Corresponding experimental data have been rare up to now, as most observations were mainly concerned with qualitative features of the microstructure. A detailed analysis of experimental data, on the other hand, requires geometrical models for the magnetic microstructures in these systems. Such models may provide insights on the multi-variant ground-states and their geometrical arrangement of variants and magnetic domains. However, modelling efforts regarding the field-driven transformations in the magnetic shape-memory materials have largely been restricted to thermodynamic considerations of their phase diagram, [142] corresponding simplified macroscopic constitutive models, [143] and constrained models for martensitic textures in bulk systems. [144] Macroscopic effective approaches have also been developed to describe coexistence and switching of (crystallographic) domains, e.g., by using phase theory approximations [17] or homogenization methods for polycrystals. [145] Computational micromagnetic models for martensitic textures could treat only systems with very limited length scales [146] and no detailed simulations along this way have been attempted, yet. [147]

Recently, twin boundary motion and microstructural transformation have been investigated in some detail for Ni-Mn-Ga single crystal slabs [127]. The experiments for the chosen orientation relationship between the single-variant state of the crystal slab and the driving field produces a simple microstructure through the nucleation of twins from one variant type. As we will show, these twins start to interact through demagnetization effects. The intermediate mixed state can be considered as a stripe domain structure with some aperiodicity that is related to coercivity effects. In general, with limited number of nucleation sites for the new twin variants and easily mobile twin-boundaries, the formation of a regular stripe-like structure with a fixed (constrained) periodicity is the expected result. These experiments offer considerable insight into the transformation processes as the geometry of the twinned state remains simple and controlled experiments on the transformation process can be performed.

Referring to these experimental observations, which will be summarized in the next section 4.2, a phenomenological theory is developed that allows for a qualitative and even quantitative interpretation of the variant nucleation. The redistribution and the concomitant magnetization processes are described by a geometrical model of magnetic domains and variants. The simple geometry of the standard experiments reported in Ref. [127] allows to use a relatively simple model for the magnetic microstructure, which can be viewed as one-dimensional sequence of differently magnetized stripes. The model relies on a previously developed theory for the equilibrium or ground-state stripe domain structures relevant for ferromagnetic martensitic plates [V]. Using the simplification that there are no important effects due to internal 180-degree magnetic domain structures, an analytical evaluation has been achieved for the micromagnetic energy of such domain models. This allows to determine detailed phase diagrams for stable and metastable states and calculation of their characteristic lengths.
This approach is extended by treating the nucleation of isolated twin-variant plates and the field-evolution towards a regular stripe domain structure with constrained periodicity. The comparison of the results from the domain model with the observed magnetization processes reveals a reasonable agreement.

### 4.2 Experimental observations

Here, we briefly summarize experimental results [148] relevant to our model.

Optical images of the sample surface at different field strengths are shown in Fig. 4.2. Before the application of the magnetic field, a nearly single variant state has been created by applying large fields in the $x$ direction. Then, the nucleation of new variants and the transformation processes is driven by a field $H$ applied in $z$ direction.

At zero field the sample is in a single variant state with a small amount of residual variants (oblique dark lines in Fig. 4.2 (a)) with the $c$-axis aligned along the $z$-direction, which also is the easy magnetization direction of these tetragonal twins. An increasing field is then applied in the $z$ direction, i.e. parallel to the $c$-axis of the residual variants. Up to a certain threshold field of about 175 mT, the variants with $c$-axis parallel to the field slowly increase in their width (Fig. 4.2 (b)). On further increasing this field to 180 mT, these variants grow abruptly (Fig. 4.2 (c)). Within a small field-interval, the twin boundaries sweep large parts of the material in a jump-like movement that is unobservable by the used quasistatic observation technique. Further increase in field leads to gradual growth of the new variants.

Fig. 4.3 shows the results of macroscopic measurements of magnetization and strain in the same single crystal with the same orientation and prehistory, i.e. starting with nearly single-variant state. The $m(H)$ hysteresis loop is shown in the main panel. The initial magnetization loop is enlarged in Inset (B). The strain as a function of the magnetic field for the first hysteresis loop driving the transformation processes is plotted in Inset (C). After applying the maximum magnetic field perpendicular to easy axis of the initial single variant the expected strain of 6% for a full transformation of this Ni-Mn-Ga single-crystal is observed. After completion of the transformation, the strain stays constant over the full magnetization/demagnetization cycle. The initial state of the sample is reached again by a mechanical load. The threshold character of the transformation process between single and multi-variant phases is also observed in the magnetization (Fig. 4.3 Inset (B)) and strain curves (Inset (C)) versus the applied field.

Summarizing the experimental observations, we find that, in an almost single-variant state of a trained magnetic shape-memory single-crystal, there are initial nucleation sites for twin variants. These nucleated twin seeds can be activated by an applied magnetic field. At a threshold field, a rapid growth of these variants takes place, that yields a more or less regular stripe-like two-variant state.

### 4.3 Phenomenological model

To model the observed magnetic multi-variant state, we set up a simplified domain-model consisting of magnetized variants that are separated by twin-boundaries with a crystallographically fixed orientation. Magnetic shape memory alloys are characterized by a rather...
Figure 4.2: (Color) Magnetic field driven reorientation transition in Ni$_{1.98}$Mn$_{1.97}$Ga$_{0.95}$ single crystal with dimensions 24×5×2.5 mm$^3$ [148]. Structural contrast of the sample surface obtained in an optical polarization microscope. At zero field the sample is in a single variant state with the magnetization parallel to the surface. The oblique lines or planes with darker contrast (orange in the schematic presentation on right side) are the nucleation sites of new variants (a). These variants have their c-axis, i.e., the easy magnetization axis along the z-direction. The (bright) regions are the remaining regions with the structure of the initial or old single-variant state with easy magnetization axis in x-direction. The field is applied in z-direction perpendicular to the sample edge.
Figure 4.3: (Color) Experimental magnetization and strain curves for the same Ni\textsubscript{1.98}Mn\textsubscript{1.07}Ga\textsubscript{0.95} single crystal as in [148]. Steep regions in both curves correspond to the threshold transitions between the single and multi-variant states at critical fields $H_1c$ and $H_2c$. Insets: (A) Sketch of a multi-variant pattern based on the observations in Ref. [127]. (B) Details of the magnetization curve. (C) Strain versus applied field curve.

strong magnetocrystalline anisotropy. This fixes the magnetization vectors along the easy directions of magnetization. Therefore, there is a “one-to-one correspondence” between magnetic domains and martensite variants in the martensitic microstructure. [130,131] This allows to describe magnetic field-driven evolution of multi-variant states within the regular phenomenological theory of magnetic domains. [2] As a model we consider multi-variant states in a layer with a thickness $T$ along the $z$ axis and infinite extension along the $x$ and $y$ directions (Fig. 4.4). In the elastically compatible martensitic microstructure, the twin-boundaries between the different variants have a fixed orientation. Assuming the geometry of the experiments reported in section 4.2, the twin-boundaries between the twin variants with tetragonal axis (and magnetically easy axes) along the $x$, respectively $z$ axis, are inclined approximately under 45 degrees in the $xOz$ plane (Fig. 4.2). For simplicity it is assumed, following Refs. [17,143], that the martensite in Ni-Mn-Ga systems can be modelled by three variants for a (pseudo)tetragonal lattice structure. The actual crystal structure of the modulated Ni-Mn-Ga martensite states has lower symmetries [149–151].

The approximation of a pseudotetragonal twinning with 45-degree orientation of the twin boundaries is sufficient to describe the simple 2-variant microstructures observed in the experiments on well-trained single crystals. However experimentally observed patterns can also include microstructures built from two coexisting single-variant states with straight interfaces, i.e. the angle between easy-magnetization axis and $z$ is equal to 0 (Fig. 4.4 c). The orientation of these interfaces with 90-degree magnetic walls are fixed by the crystallography of the twin-boundaries between the martensite variants. We discuss both cases with oblique and straight interfaces.

Many experimentally observed multi-variant patterns include a number of tetragonal variants with internal magnetic domains separated by 180-degree domain walls (see Fig. 4.3 Inset
Figure 4.4: (Color) Multivariant patterns consisting of two different tetragonal martensites: with an isolated stripe (a) and with a system of stripes with oblique interfaces (b) and straight interfaces (c). The nucleated new variants have magnetization which is favorable for the magnetic field $H$ applied perpendicular to the plate, in $z$-direction.

(A) and Ref. [127]). Here, however, we consider a simplified model with tetragonal variants homogeneously magnetized along $x$ and $z$ directions. As a result, the simplified model assumes that effects of internal magnetostatic charges along the twin-boundaries are weak or absent. This assumption remains satisfactory, if the 180-degree domain structures within a single twin, as shown in Fig. 4.3 Inset (A), is fine enough so that demagnetization effects of these variants are effectively cancelling, or if the 180-degree domain structure remains also magnetically compatible across the twin-boundaries by avoiding the occurrence of magnetic poles.

In the experimentally investigated sample a large enough magnetic field applied along the long dimension (the $x$ axis) creates a nearly single variant state with the magnetization aligned with the field. As discussed in the section 4.2, there remain nucleation seeds as oblique planes with a fixed location in the sample (Fig. 4.2 a)). Under an increasing magnetic field applied along the $z$ direction these variants grow, form a stripe-like structure, beyond a threshold field ($H > H_c$), and then gradually expand into the saturated state. Because the observed multi-variant states are formed by the expansion of the “nucleation” planes (Fig. 4.2 c) the geometrical arrangement of these multi-stripe patterns is pre-determined by the location of these nucleation seeds. The spatial distributions of the lines is generally aperiodic, but it can be characterized by the “period” $D$ that equals the average distance between the neighbouring lines.

Therefore, we describe the experimentally observed multi-variant states (Fig. 4.2) by a model of stripes with a fixed period $D$. Depending on the relation between the new variant size $d$ and the period $D$ the new variants can be treated either as isolated ($d \ll D$) or interacting objects. In the following we investigate both situations.

### 4.3.1 Multi-stripe patterns

The patterns in Fig. 4.4 on the one hand satisfy the elastic compatibility between the tetragonal variants and, on the other hand, comply with a common property of magnetic domains by avoiding uncompensated magnetostatic charges on the domain boundaries. We assume that the uniaxial anisotropy is much stronger than the applied fields. Thus, deviations of the magnetization from the easy axis within the variants can be neglected. As sketched in Fig. 4.5, the magnetization in the microstructure with oblique twin interfaces can be reduced
Figure 4.5: (Color) Multivariant patterns consisting of two different tetragonal martensites, \( \beta = \pi/4 \). The system can be decomposed into two interacted subsystems: homogeneously magnetized layer with saturation magnetization equal to \( M_s/2 \) and stripe domains with alternating magnetization perpendicular to the surface, \textit{oblique} domain walls and surface magnetic charges density \( \sigma = M_s/2 \).

to a pattern with charge density \( \sigma = \tilde{M}_s = M_s/2 \) perpendicular to the plate surface and an effective bias field \( \tilde{H} = H_z + 2\pi M_s \). For the pattern with straight interfaces in Fig. 4.4 c, the alternating magnetization \( \sigma = \tilde{M}_s = M_s/\sqrt{2} \), and the bias field is \( \tilde{H} = H_z \). The model with straight domain walls has been discussed in details for magnetic films in Sect. 1.7.1 and the solutions for the micromagnetic problems can be applied directly for magnetic shape memory films.

The energy density \( e_{tot} = E_{tot}/(2\pi \tilde{M}_s^2) \) for an oblique periodic stripe structure with \( \beta = \pi/2 \) first has been derived in Ref. [V]. It can be written

\[
e_{tot} = \frac{2\sqrt{2}p l_c}{\pi} - \frac{\tilde{H}}{2\pi \tilde{M}_s} q + e_m. \tag{4.2}
\]

Here, the first term is the domain wall energy, the second one is Zeeman energy and the last one is magnetostatic energy. In Eq. (4.2) the inverse reduced period \( p = 2\pi t/D \) and the fraction of the new variant \( q = (2d - D)/D \) are the internal variables of the system. Note that parameter \( q \) and reduced magnetization \( m = M/M_s = d/D \) are limited by the relation \( m = (1 + q)/2 \) for oblique and \( m = q \) for the straight domains. The magnetostatic problem of oblique stripe domains has close relation to the problem of shifted stripe domains discussed in Sect. 2.2. In particular, according to the Fig. 4.5 the magnetostatic energy of such stripe domains with uncharged oblique domain walls can be written as interaction energy between two surfaces with stripe-like charge distribution. The charge distribution in upper and lower surfaces are shifted in \( x \)-direction by \( a = t \cdot \tan(\beta) \) (see Fig. 4.5). Then, according to Eq. (1.16) the magnetostatic self energy density of a single magnetic layer with thickness \( t \) and oblique stripe domains at an angle \( \beta \) can be written as

\[
E_m = \int_0^D (\sigma_s(x)\phi_s(x,0) - \sigma_s(x)\phi_s(x-t \cdot \tan(\beta),t)) \, dx, \tag{4.3}
\]

where \( \phi_s \) and \( \sigma_s \) are defined by Eqs. (1.44) and (1.41), respectively. After integration, the stray field energy \( E_m \) for the case of \( \beta = \pi/2 \) is given by the expression [V]

\[
E_m = 2\pi \tilde{M}_s^2 \left( q^2 + \frac{4}{\pi^2 p} \sum_{n=1}^{\infty} \frac{(1 - (-1)^n \cos(\pi n q))}{n^3} \left[ 1 - \cos(np \tan(\beta)) e^{-np} \right] \right). \tag{4.4}
\]

With the help of an integral transformation similar to those given in Sect. 2.2 the infinite
Figure 4.6: (Color) Solutions for the widths of isolated ($d_s$) and interacting ($d, d^*$) stripes as functions of the applied field in a plate with thickness $t/l_c = 16$. For $0 < H < H_c$ the isolated new variant exists as a metastable state. In the interval $H_{1c} < H < H_{2c} = (4\pi M_s - H_{1c})$ the multi-variant stripe phase is the ground state of the system. The inset shows the phase diagram for multi-variant stripes with periodicity $D$ versus layer thickness $t$. Stripe states can exist only above the critical line $D(t)$ (white area), while below the critical line (shaded, gray area) no stable or metastable multi-stripe states are formed.

The sum in Eq. (4.4) is transformed into integrals on the interval $[0,1]$. Then, the energy (4.2) can be written in closed form

$$e_{\text{tot}} = 1 - \frac{4p}{\pi^2} \int_0^1 (1 - \xi) \arctan \left[ f(\eta, q) \right] d\xi + \frac{p t_0}{t} - 2q \left( \frac{H}{2\pi M_s} - 1 \right), \quad (4.5)$$

where

$$f(\eta, q) = \frac{\sinh \eta \sin \eta \left[ 1 + \cos(\pi q) \right]}{\cosh^2 \eta - \cosh \eta \cos \eta - \sin^2 \eta + (\cosh \eta \cos \eta - 1) \cos(\pi q)}. \quad (4.6)$$

Here, the rescaled lengths parameters $t_0 = 2\sqrt{2}l_c$ and $\eta = pt$ have been introduced. Minimizing the energy density (4.5) with respect to the internal parameters $p$ and $q$ yields the equations for the equilibrium sizes of new and old variants $d$ and $d^* = D - d$, respectively, in a periodic multi-variant stripe pattern (see Fig. 4.6). For straight stripes ($\beta = 0$) the integral transformation of sum (4.4) is coincident with Eq. (1.47), Sect. 1.7.1.
4.3.2 Isolated new variants

The magnetic energy of an isolated oblique domain with a width $d$ in a plate of thickness $t$ (Fig. 4.4) includes the Zeemann energy from the field $H$, the dipolar stray field energy $e_m$ and the domain wall energy along twin boundaries, which is proportional to the domain wall (areal density) energy $\gamma_{tb}$. In this approach we assume that the domain wall energy $\gamma_{tb}$ does not depend on magnetization distribution within the variants and includes contribution from the magnetic interactions (as anisotropy and exchange) as well as contribution from the elastic energy. Per unit lengths in $y$-direction, total magnetic energy can be written as

$$E = 8\pi M_s^2 l_c t - H M_s t d + M_s^2 t^2 e_m,$$  \hspace{1cm} (4.7)

In the first term, considering the domain wall energy, we introduce the characteristic length $l_c = \gamma_{tb} / (4\pi M_s^2)$, which is the sole materials parameter determining the domain structure in this model. The stray field energy contribution $e_m$ can be reduced to the magnetodipole energy of two “charged” stripes and can be derived following the procedure introduced in Ref. [2]

$$e_m(\tau) = f_{(+)}(\tau) + f_{(-)}(\tau) - 2\tau^2 \ln(2\tau^2) - \pi,$$  \hspace{1cm} (4.8)

where $\tau = d / \sqrt{2t}$ and

$$f_{(\pm)}(\tau) = \tau(\tau \pm \sqrt{2}) \ln \left[2(\tau^2 \pm \sqrt{2}\tau + 1)\right] + 2(\sqrt{2}\tau \pm 1) \arctan(\sqrt{2}\tau \pm 1).$$  \hspace{1cm} (4.9)

Minimization of the energy $E$ in Eq. (4.7) yields the following equation

$$H/M_s = \sqrt{2}\tau \ln \left(1 + \tau^{-4}\right) + \ln \left(\frac{\tau^2 + \sqrt{2}\tau + 1}{\tau^2 - \sqrt{2}\tau + 1}\right) + 2 \arccos \left(\frac{1 - \tau^2}{\sqrt{1 + \tau^4}}\right),$$  \hspace{1cm} (4.10)

which gives the solution for the equilibrium size of the new variant $d_s(H)$ in an implicit form.

4.4 Results and Discussion

Typical solutions for the width of isolated variants $d_s(H)$, and for the variants in a periodic stripe pattern $d(H)$, and $d'(H)$ are plotted in Fig. 4.6. The analysis shows that for all equilibrium values of $p > 0$ and $0 < q < 1$ the energy of the multi-variant states is lower than that of the single variant state [V]. The stripes are transformed into the single variant phase continuously by an unlimited growth of the period, $p \to 0$. However, at the transition line the other, coexisting variant has a finite width. The solutions for a new isolated variant exist in the interval $[0, 2\pi M_s]$ (Fig. 4.6). As the applied field varies in this interval, the size $d_s$ of the nucleated variant monotonically increases from zero at $H = 0$ to infinity at $H = 2\pi M_s$.

The condition for equilibrium between the homogeneous state and the state with an isolated new variant is determined by the equation $E = 0$ for the energy in Eq. (4.7). Combining this
equation with Eq. (4.10) we obtain

\[
\frac{l_c}{t} = \sqrt{\frac{2}{\pi}} \left[ \tau^2 \ln (1 + \tau^{-4}) + 2 \arctan (\tau^2) \right].
\]  

(4.11)

Eqs. (4.10) and (4.11) determine the critical field \(H_c(t)\) (Fig. 4.7), respectively the critical width \(d_c(t)\) (Fig. 4.6) of the isolated stripe. For \(H < H_c(t)\) the isolated stripe has a positive energy (with respect to the homogeneous state). In decreasing field \(H < H_c\) the stripe gradually shrinks to zero size at \(H = 0\).

For \(H > H_c\) the energy of the isolated stripe is lower than that of the homogeneous state. Then, the nucleated stripes condense into a one-dimensional lattice of interacting stripes along the \(x\)-direction. This means that the critical field \(H_c\) is a transition field between the single-variant and multi-variant phases. In the magnetic phase diagram (Fig. 4.7) these two phases are separated by the critical line \(H_c(t)\). This line ends in the point \((t_0, 0)\), where \(t_0 = 2\sqrt{2}l_c\) is the critical thickness. In samples with \(t < t_0\) multi-variant states are always energetically unfavourable. In such systems a magnetic field may drive a switching process directly from one single variant state to another, under the condition that coercivities do not lock-in metastable states.

It has to be stressed that the existence of the critical thickness is a specific effect imposed by the particular geometry of multi-variant states with the oblique twin boundaries oriented under 45 degrees in the present model for shape memory alloy plates. On the contrary, in layers of common magnetic materials with perpendicular magnetization and straight domain walls, the usual striped multi-domain states always are energetically favourable. In the case
Figure 4.8: (Color) The phase diagram for plates with thickness $t$ in applied field $H$ for stripes with different fixed periods $D$. Regions for multi-variant stripes with fixed period $D$ exist only above a critical layer thickness, compare inset in Fig. 4.6, in a limited field range. The critical line for $D = \infty$ corresponds to the equilibrium stripe states, that reach infinite period lengths at the critical fields, see Fig. 4.6.

of such perpendicular magnetic layers, the stripe domain structure (theoretically) exists for any thickness [2, 22] (see also discussion in Sects. 1.7 and 2).

In the whole range of their existence the solutions for isolated variants are metastable. Below $H_c$ they are metastable with respect to the homogeneous initial variant state, at higher fields ($H_c < H < 2\pi M_s$) with respect to the multi-variant phase. According to the ground-state structure of the equilibrium solutions for multi-variant states, new variants should arise at the critical field $H_c(t)$ as isolated twins with the finite size $d_c(t)$, i.e., the critical size equals the finite size of the minority variant in the stripe states. Hence, exactly at $H_{1c}$ the nucleated new variants in the stripe pattern has the same critical size as the isolated metastable variant immediately below $H_{1c}$, see Fig. 4.6. As the applied field increases to $H = 2\pi M_s$ these twins would condense into a regular one-dimensional lattice of multi-variant states. For $H > 2\pi M_s$ the twins with the new variant structure would become larger than those remaining twins with the old initial variant structure. At the critical field $H_{2c} = 4\pi M_s - H_{1c}$ the multi-variant phase finally is transformed into the homogeneous state by an unlimited growth of the new variant size $d$. In real systems, however, the equilibrium states can rarely be reached.

In the Ni-Mn-Ga single crystal slab shown in Fig. 4.2 the number and positions of new variant domains apparently are predetermined by the separation of nucleation centers in the sample. Disregarding the aperiodicity of the sites for these nucleation planes, we introduce an averaged period $D$ in our model. Then, a magnetic-field-driven evolution of multi-variant states in Ni-Mn-Ga sample can be described with a fixed period $D$. This situation is described by Eq. (4.5) with the parameter $p$ maintaining a certain fixed value. Then the fraction $q$ of twin-variants per period lengths is the only variable of the system. The equation $de_{\text{tot}}/dq = 0$
yields the equation for the equilibrium values of $d$

$$H = 2\pi M + 4M_p \int_0^1 (1 - \xi) g(\eta, q) \, d\xi,$$

(4.12)

$$g(\eta, q) = \frac{2 \sin(\pi q) \sinh \eta \sin \eta}{4 \cos^2(\pi q/2) \left[\cos^2(\pi q/2) + \cosh \eta \cos \eta - 1\right] + (\cosh \eta - \cos \eta)^2}.$$

(4.13)

The condition of equilibrium, Eq. (4.12), for the the constrained model with fixed period $D$ does not contain the characteristic length $l_c$ that depends on the magnetic wall energy $\sigma$. This is easily understood, as the density of interface energy per lengths is fixed by $D$. Thus, the fraction of the new variant in such a state only depends on layer thickness $t$ and magnetization $M$. The calculated phase diagrams for systems with fixed periods are plotted in Fig. 4.8. Compared to the plates with the equilibrium periods these systems have larger values of the critical thickness and narrower field intervals for the (meta)stable multi-variant states. The smaller the period the smaller is the interval for the occurrence of multi-domain states in the phase diagram (Fig. 4.8). The corresponding magnetization curves (Fig. 4.9) show magnetization jumps in the transition fields, which directly translate into a jump-like strain evolution. Finally, multi-variant states cannot exist with fixed periods that are smaller than the minimal solution for $D$, i.e., the equilibrium period at $H = 2\pi M$. For such cases within the model, the competition between the single-variant states and the multi-variant state with a too short period results in a direct transition between the two single-variants at the critical field $H = 2\pi M$. Physically, this means that the transition is achieved in a single jump, if the number of nucleation planes for the new variant is very high in the crystal.

The model results compare well with the experimental observations. The strains plotted in Fig. 4.3 Inset (C) are approximately proportional to the new variant fraction $q(H)$, and correspondingly to the magnetization along the $z$ axis. The calculated magnetization and strain curves (Fig. 4.9) are in a good qualitative agreement with these experimental results.

The model correctly describes the transition into a multi-variant state by a threshold effect as seen in the experiment on a single crystal of the Ni$_{1.98}$Mn$_{1.07}$Ga$_{0.95}$ magnetic shape-memory material (Figs. 4.2, 4.3) [148]. New variants are formed by a sudden expansion of the “nucleation planes”, which are pre-formed martensite twins, at certain critical field. The further evolution of the multi-variant state occurs by expansion of the existing new variants (without the formation of new variants). The phenomenological model (Eqs. (4.7) and (4.5) gives a basic micromagnetic description of this magnetization process.

The simplicity of the present model allows a rigorous analytic evaluation of the dipolar stray field energy. The results show that the magnetization processes rule the rearrangement of twin-variants in a single-crystal plate. To achieve a quantitative model, future work needs to address two refinements: (i) The occurrence of 180-degree internal domain structures and internal magnetostatic charges along twin boundaries, as indicated in Fig. 4.3 (A) (see Ref. [127]). (ii) The rotation of magnetization within the magnetic domains owing to finite magnetic anisotropy. The rotation of magnetization in the initial variant is clearly seen in the initial susceptibility of the experimental magnetization curve. In our model, we do not consider these deviations of the magnetization in the old variants by employing the limit of
Figure 4.9: (Color) The magnetization and strain curves for stripes with fixed periods indicate magnetization jumps during the transformation into a multi-variant states, which is a magnetic multi-domain state. The magnetization (left scale) and strains (right scale) are proportional to the new variant fraction $q(H)$.

In conclusion, the model results reveal the dominant role of dipolar magneto-static interactions for the formation and evolution of the multi-variant states. In trained magnetic shape-memory materials a sufficient, but small number of crystalline defects as stacking faults or already nucleated twin-plates do apparently exist. This means the applied external field does not need to overcome energy barriers of the order of chemical bonding to create new lattice defects and eventually new twin planes. Instead, the magnetic nucleation of new twin variants is essentially determined by the resulting magnetization distribution and the macroscopic demagnetization effects. On the other hand, the observations also indicate that no new twin variant planes will be nucleated in the lattice by the applied magnetic field, as is understandable from the high energies required to create such extended lattice defects as a twin plate. Therefore, the microstructure and the evolution of the transformation in these materials is likely determined by the existing defect structure and could be tailored by design of defects. In our theoretical treatment, we have considered this extrinsically determined microstructure by fixing the average period of the stripe structure as the model for this multi-variant structure.
Conclusions and Outlook

In this work we have presented theoretical studies of multidomain states in antiferromagnetically coupled multilayers and ferromagnetic shape memory alloys. Our theoretical approach is based on domain theory, the main principles of which are given in the first chapter of the thesis. We used an original method for the evaluation of geometrical models of stripe and bubble domains in magnetic single and multilayers. This method is based on rigorous solutions of the corresponding magnetostatic problem by using an integral representation of magnetostatic energy instead of infinite sums. Because of the particular nature of high perpendicular anisotropy multilayers the domain size in these systems is much bigger than the thickness of the nanoscale multilayer. Due to this feature the series expansion of the magnetostatic energy is slowly convergent. The integral representation of the magnetostatic energy allows to avoid this problem. In particular, for the case of magnetic multilayers our approach permits to simplify general expressions for the total energy of the system. The resulting closed expressions also are easily evaluated by means of numerical calculations.

Concerning the part of the thesis which is devoted to the antiferromagnetically coupled multilayers with strong perpendicular anisotropy the results can be summarized as following. We have demonstrated that, in antiferromagnetically coupled multilayers, ferromagnetic stripes are unstable with respect to a lateral shift of domain walls in adjacent magnetic layers. These multidomain configurations form *shifted ferro stripe* states which are stable (corresponding to the global or local energy minimum of the system) only in a certain range of the magnetic layer thickness and the interlayer exchange coupling. Below some critical thickness of the magnetic layer, which generally depends on interlayer exchange coupling strength and geometrical parameters, the multilayer system transforms into either homogeneous antiferromagnetically ordered state (at small exchange) or into an antiferromagnetic multidomain state (at high exchange value). The latter state is a new theoretical prediction. Antiferromagnetically ordered regular multidomain (antiferro stripes) states have not been observed up to now because of comparably high interlayer exchange value needed to achieve it. However, as it was shown in Chapter 2, an antiferromagnetic multidomain state should exhibit properties which are quite different compared to known ferromagnetic domains and could be promising objects for future investigations. In particular, contrary to ferromagnetic domains antiferro stripes are stable with respect to lateral shift. Also antiferro stripes transform by a continuous transition into a homogeneous state at a definite critical thickness that is solely determined by geometrical parameters. The study of antiferro stripes in external magnetic field also could be interesting from both theoretical and experimental point of view.

In Chapter 2, we have presented a detailed theoretical analysis of domain structures in antiferromagnetically coupled perpendicular multilayers with homogeneous antiferromagnetic ground state. The model development has been motivated by a series of experiments on such synthetic metamagnetic systems. A thorough qualitative and even quantitatively satis-
factory comparison with experimental data on \{[\text{Co/Pt}]_n\} systems has been achieved by this model. Our findings reveal a large variety of possible multidomain states in AF coupled multilayers with perpendicular anisotropy. The complex evolution of the specific (metamagnetic) multidomain states induced by external fields elucidates reorientation effects and the formation of isolated stripe and bubble domains within the saturated states and antiferromagnetic remanent state. Direct observation of metamagnetic domains confirms the theoretical description of this evolution. Within the micromagnetic approach metamagnetic domains can be described by a modified model of ferromagnetic domains. This has allowed to derive the equilibrium parameters of metamagnetic stripe and bubble domains and calculate magnetic phase diagrams. These diagrams for the field-driven equilibrium states can also provide the basis for future investigations of the hysteretic processes induced by coercivity and the dynamics at the various magnetic phase transitions in these multilayer systems. Future theoretical investigation should also address models which take into account possible noncollinear configurations which can appear in the case of weaker perpendicular anisotropy.

In Sect. 2.4 an exhaustive analysis of specific topological defects (ferrobands) arising in perpendicular antiferromagnetically coupled multilayer is presented. Magnetic-field-driven evolution and transformation of these defects explain the formation of remanent states recently observed \([\text{Co/Pt}]_n\)Ru and \([\text{Co/Pt}]_n\)NiO antiferromagnetic multilayers. According to experimental observations ferrobands can exist either in a single domain state or split into a system of domains creating, so called “tiger-tail” patterns [104]. Future investigation should address the stability of these multidomain states. Further experimental investigations of “tiger-tail” patterns together with a micromagnetic analysis of these multidomain patterns should give deeper insight into the formation and evolution of topological defects in this class of magnetic nanostructures.

The model of stripe and bubble domain in magnetic multilayer mentioned above can also be applied to calculate stray fields above these multilayer system that are observable by magnetic force microscopy (MFM). In Chapter 3, it is shown that the ground-state ferro and antiferro stripe structures in antiferromagnetically coupled multilayers differ by their period lengths and by the spatial distribution of their stray fields. Our analytical calculations executed within a simplified model of one-dimensional multidomain patterns with fixed magnetization orientation and infinitely thin domain walls can be extended to more realistic models which take into account finite width of domain wall and can be employed to model general features of MFM images from perpendicular multilayers.

The last part, Capter 4, of the thesis is devoted to the investigation of ferromagnetic shape memory materials. Our results reveal the dominant role of dipolar magneto-static interactions for the formation and evolution of the multi-variant states. Our theory enables rigorous calculations of equilibrium structures in thin ferromagnetic martensite plates with two variants. We find that oblique stripe structures are possible only above a certain critical thickness, while straight stripe structures have been known to exist for arbitrary dimensions of layers with perpendicular magnetic anisotropy.

It is clear that real magnetization processes in magnetic shape-memory processes will generally be hysteretic. The microstructure and the evolution of the transformation in these materials is likely determined by the existing defect structure. In our theoretical treatment, we have considered this extrinsically determined microstructure by fixing the average period of the stripe structure as the simplest model for this multi-variant structure. Future investigations should include effects of magneto-elastic interactions which also play an important role.
in this system. Furthermore, micromagnetic model should be extended to treat martensite variants with arbitrary angle between easy axis and magnetization.
List of original publications

This thesis is based on the following articles, which are referred to in the text by Roman numerals ¹:


¹Nikolai S. Kiselev is an alternative transcription of my name.
References


References


References


References


References


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