The Best of Both Worlds: Teaching Middle School and College Mathematics
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Abstract
As a full-time Professor of Mathematics Education, as well as a part-time eighth grade (13 and 14 year olds) mathematics teacher, I have the opportunity to experience the teaching profession from “both sides of the fence.” My university courses are enhanced by my work in the field, while my eighth graders’ learning is strengthened by educational principles studied at the university. In this paper (and presentation), I will explain this partnership and the benefits to both audiences.

Introduction
For the past 15 years, I have taught mathematics and mathematics education courses at Bowling Green State University (BGSU) in Bowling Green, Ohio (USA) as my full-time responsibility. However, I have also taught one middle school (Grade 8) mathematics course at a local school each day during the same time period. I teach at St. Rose School in Perrysburg, Ohio (USA) – a Catholic school with approximately 400 students. My typical class size has been about 20 students per year.

I have found that my experiences in the middle school classroom feed and enhance my teaching at the college level, as well as my ability to make presentations and author books and journal articles. Additionally, my eighth graders have benefited from my sharing with them about the nature of college mathematics – what they will be expected to know, which of the “big ideas” in our class will be expanded upon later, and so forth. When making presentations to teachers and administrators, I always share real examples of teaching strategies I use with my own secondary school students, including challenges that arise and opportunities that present themselves.

Dividing Responsibilities
When I was hired at our university (BGSU) in 1994, there was an agreement that I would be permitted to continue teaching eighth grade mathematics, which I had been doing for five years prior to beginning my college teaching career. My eighth grade class started at 7:15 a.m. each day, Monday through Thursday (ending at 8:15 a.m.), and the university agreed to allow my teaching, office, and committee responsibilities to begin late enough that I could get to campus in time. Consequently, I generally never teach a class that begins before 10:00 a.m. and schedule all meetings and office hours for 9:00 a.m. or later. This agreement has held for 15 years and makes it possible for me to teach in both settings.

St. Rose School pays me to teach the class, although the salary is minimal. A colleague of mine who lives in another state teaches in a local school in a similar manner. However, in her case, rather than the school paying her to teach, the school district transfers money to the university, who uses the funds to buy her out of a course-teaching load. Therefore, her “in load” of teaching includes two university courses and one middle school class, as opposed to teaching three classes on campus. This arrangement has not been feasible at my institution, so I teach three courses on campus each semester, in addition to the eighth grade class. I estimate that I spend approximately 10 hours per week on planning, teaching, and grading papers at St. Rose. I accomplish this work by rising early in the morning and sometimes work well into the evening. I do not allow my eighth grade teaching time to interfere with my productivity at the university and try to keep the two separated. In reality, both positions feed and enhance one another, and I am more productive in both settings with this arrangement.

Seven years ago, I took a sabbatical from BGSU and was hired as a mathematics teacher at a local high school. For that academic year (2001-2002), I taught high school mathematics on a full-time basis, including teaching five classes across the grade levels (Grades 10-12), co-chairing the Mathematics Department, and doing other school-related work, such as chaperoning or supervising student events. My intent was to immerse myself into the school setting for a full year to enhance my ability to teach education courses upon returning to the university. Similarly, I am on leave for the autumn of 2009 and am again teaching in schools. I am teaching my eighth grade class, along with teaching a statistics course in a local high school.
Issues Raised in Classes

One day, in the fall of 2008, I was teaching my eighth grade class how to solve linear equations. The equation on the board was: \[ 5 - 2(x + 4) = x + 6. \] One of my students went to the board, subtracted 2 from 5 (ignoring the correct order of operations), distributed the “3” through the parentheses, and obtained an answer of \( x = -3 \).

Another student raised her hand and said, “That can’t be right. He forgot to distribute the \(-2\) through the parentheses.” She went to the board, followed the “correct” procedure, and also found a solution of \( x = -3 \). Of course, a third student raised his hand and asked, “So, does it matter whether you distribute first or not? It looks like you get the same answer either way!” Suddenly, I found myself having to defend why the proper use of order of operations would always result in a correct answer and why this particular problem was an exception, rather than a rule.

The next morning, I went into my mathematics teaching methods course on campus, put the problem on the board, explained what had happened, and asked my students two questions:

1. How would you have handled the eighth grader’s question?
2. In what cases would we expect the answer to be the same, whether the distributive property is used or not?

The first question got us talking about how students learn mathematics and the pedagogy that can be used to meet students’ needs, while the second question allowed us to delve more deeply into the mathematics of the problem.

Eventually, we graphed the functions \( y_1 = 5 - 2(x + 4) \) and \( y_2 = 3(x + 4) \), as well as \( y_3 = x + 6 \) and showed that, by coincidence, all three of these linear functions intersected at the point \((-3, 3)\). Since all three of the lines share the same intersection point, both ways of working the problem will result in the same answer. A counter-example had to be used to prove that this is not the case for a similar equation, such as \( 5 - 2(x + 4) = x + 5 \). My students went on to explore the conditions under which both the “right” and the “wrong” processes would always result in the same correct answer.

This is just one example of a situation that arose in my eighth grade class that I could take back to my university students. On another occasion, an eighth grader asked me, “I know that a line has a slope, but does a parabola have a slope too?” Again, I took this question back to my methods students at the university, and we discussed how to get a secondary school student to understand the basic idea of “slope of a tangent line,” which eventually develops a fundamental idea of calculus.

Similarly, one day in my university class, a student revealed a misconception about what it means for a weather forecaster to say that there is a “20% chance of rain.” Several of my college students thought it meant that 20% of the area would receive rain today, while the other 80% would not. I went back to my eighth graders and asked them what they believed the prediction meant. It made for an interesting contrast of thinking, as it turns out that my 14-year-olds understood the notion of probability better than many of my university students! The situation helped me to understand the thinking of both levels of students, which in turn makes me a better instructor.

Problems Used with Students

One of my favorite problems I use with my students at both levels is called the Orange Grove Problem. Students are presented with a scenario in which an orange farmer has a grove of 120 trees in which each tree averages 650 oranges per season. But if he plants additional trees, each tree takes up enough nutrients from the soil that the average production of every tree decreases by 5 oranges. The question is how many trees to plant to maximize the orange production. What I find interesting about the problem is how students at different levels solve it. For example, most eighth graders will make a table and then look for a pattern. So, if one tree is planted, the grove has 121 trees, but each produces an average of 645 oranges. The second tree results in a total of 122 trees with 640 oranges apiece, and so on. By multiplying the number of trees by the average number of oranges per tree, students can see the pattern and determine that planting five trees maximizes production.

At the college level, I find students more likely to find and solve equations. Typically, students will write an equation such as \( y = (120 + x)(650 - 5x) \), where \( x \) represents the number of new trees planted, and \( y \) stands for the total orange production. They will either graph the function on a calculator and identify the vertex or multiply the binomials and take a first derivative to determine its maximum value. In the end, they also determine that planting five trees maximizes the production, but the approach is much more technical and makes use of higher mathematical skills.

In each case, once students have finished solving the problem, I show them how the other group typically deals with it. The eighth graders look at the parabola and discuss the visual approach to seeking a maximum, while college students are asked to “step back” and look at the problem as someone would view it with no calculus and very little exposure to quadratics. The experience of sharing the problem across grade levels makes the process of solving the problem much richer for everyone involved. I frequently try to pose problems that can be solved both in a
secondary and in a university setting to deepen my understanding of how students handle the same situation with different backgrounds and problem-solving tools.

**Lesson Learned**

Probably the most important lesson that I have learned through this teaching arrangement is how important it is for a university professor to remain up-to-date with what is happening in the field. So often, university students complain that their education professors “have never been a ‘real’ school teacher” or that it has “been so long since he/she was a teacher, that he/she is out of touch with schools.” I believe it is essential that university education faculty maintain a presence in school settings. While on sabbatical, for example, I had to learn the system for taking attendance on the computer and uploading grades and attendance to a school Web site so that parents and students could check their grades every day. These tools were not available when I was a full-time teacher, so it was important for me to put myself into the situation to learn what teachers are up against.

There are several avenues that can lead to a high level of exposure to teaching schools, such as the following:

1. The situation described in this paper is one way to remain in a school setting. If a local school is looking for a part-time teacher, an arrangement with a university faculty member may be possible.
2. The sabbatical experience also discussed here is another way for a university professor to be immersed in a school setting. Some institutions actually require education faculty to get out into schools to teach every few years.
3. Grants or other collaborative ventures can allow university faculty to interact directly with students in a school setting. In some cases, a grant project allows for the faculty member to teach model lessons, to tutor small groups of students, or to team teach lessons with mathematics teachers.
4. Another initiative that I have taken is to simply call a local teacher and ask him/her if you can visit his/her classroom for a couple of days and teach a sample lesson or unit. In the process of writing a journal article a few years back, I called a fifth grade teacher and arranged to spend three days teaching her class, during which time I was able to gather data, take photographs, and, most importantly, gain experience. The teacher and children were enriched by my visit, and I learned much about the day-to-day struggles (and the excitement) of being a fifth grade teacher.

**Conclusions**

I am often asked, “Why would a full-time university professor choose to spend 4 hours a week in a classroom with 24 adolescents when he doesn’t have to do it?” The answer is simple: Because I enjoy it. I chose education as my profession because I want to help children appreciate mathematics and become successful problem solvers. I believe that having an impact on these students can happen from both ends of the spectrum – as a professor who teaches others how to teach, as well as working directly with the children themselves. While I conduct most of my work at the college level, I have never lost sight of the importance of working with school-age students as well. By working in both environments, I have found my credibility level among professionals to be very high. At St. Rose School, students, parents, and administrators know that I prepare mathematics teachers, so my opinions and work are highly valued. At the university level, my students and colleagues frequently seek my opinion on issues, knowing that I am “in the trenches” every day. I was recently conducting an in-service workshop for teachers and sharing a teaching idea. One participant raised his hand and asked, “This sounds like an interesting problem in theory, but I’m not sure ‘real’ students can handle it. I wonder how it would play out in a classroom.” I replied that it was interesting he would ask that, given that I just solved that problem with my own eighth grade class within the past couple of weeks. I proceeded to show the teachers samples of how my own students had approached the problem. I try to share only those ideas that I have actually attempted myself, with my own students. As a result, I can give a presentation, write an article or book, or teach a class based on experience, rather than relying entirely on theory. I am the author of one textbook and co-author of another, and both of these were written out of my experiences in the classroom.

One major drawback of my situation is that it can be exhausting. When my eighth graders are on vacation or a class trip, I suddenly “find” 10 working hours that I didn’t have the previous week, and I find myself feeling less stressed and getting more sleep. Another difficulty I experience is with adjusting to different age groups. There are days when I teach eighth graders in the morning, college students in the afternoon, and experienced teachers in a workshop in the evening. Knowing the audience and adjusting the pace and content for the situation can be challenging. In general, however, I would not trade this experience for anything. I look forward to both components of my work and have managed to put them together in my teaching, writing, and presenting. I look forward to continuing my relationship with a local school until I retire and encourage colleagues to consider any type of arrangement that puts them in a school environment on a routine basis – not just observing teachers and children but interacting with them.