Reflections on an Initiative to Improve Junior Secondary School Pupils’ Understanding of Number
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Abstract
In 2005 the opportunity to apply the New Zealand ‘Numeracy’ approach to teaching Mathematics was extended into the secondary school sector. The goal was to alter teachers’ pedagogy so that ‘sense making’ rather than ‘instruction’ was the core objective of their lessons. Ultimately it is hoped that along with a familiarity and comprehension of Number will come a relatively seamless acquisition of the fundamentals of Algebra. This paper will present details of this approach for teaching Number, the status of Number in the secondary school curriculum, the focus and ramifications of teaching for understanding, as opposed to assimilating and learning to apply algorithms, and will also consider evidence of the effectiveness of the initiative.

Introduction
The term ‘Numeracy’ as used above refers to a person’s ‘ability and inclination to use Mathematics effectively – at home, at work and in the community’. In New Zealand ‘Number’ refers to a particular strand in Mathematics in the New Zealand Curriculum. The focus of this strand is the structure of the number system; Number Knowledge and to consider the Strategies that can be applied to find answers to problems. The Strategies become the overt reasoning that pupils use when they perform arithmetic calculations. Out of this careful consideration of the Knowledge-Strategy interrelationship it is envisaged that the transition to Algebra will be less traumatic. The connection between in-depth number knowledge, flexible mental calculations and Algebra thinking was recognized by Irwin and Britt in their study (2005).

Historical Background
When the results from the Trends in International Mathematics and Science Study (TIMSS) released in 1996 identified a degree of underachievement by a significant number of our secondary school-aged pupils, compared to the performance of similar cohorts in the global Mathematics education community the government responded by establishing a Task Force closely followed by the Numeracy Development Project (NDP) implemented in 2001 after a successful pilot of Count Me In Too, in 2000. This NDP aims to raise pupils’ achievement through teacher professional development. Exploratory work carried out in secondary schools from 2001 through to 2003 found that pupils’ achievement at year 9 level had been variable. The challenge of changing teachers’ practice at the secondary level had also been noted by a number of teachers and educators. In the evaluation of the Numeracy Exploratory Study in 2001, Irwin and Niedere (2002) noted the “unexpected weakness, especially in the understanding of fractions, the ability to find a fraction of a whole number, and the meaning of large numbers” (p 97). One factor identified as a reason was the weak pedagogical content knowledge of teachers in the area of place value. In 2002 and 2003, pupils also performed below expectations in the National Certificate of Educational Achievement (NCEA). In response to the perceived ineffectiveness of teaching Mathematics in the junior area of secondary schools the Numeracy Development Project (NDP) was extended to secondary schools as the Secondary Numeracy Project (SNP) in 2005.

Theoretical Composition of the Numeracy Project
The NDP has its roots in constructivism, the Pirie/Keiren recursive model for teaching, along with the principles of cognitively guided instruction (Holmes & Tozer, 2004). A framework detailing the aspects of staged number knowledge that pupils will require was compiled by a number of Mathematics education researchers (Frank, 1989; Fuson, 1998; Steffe, Cobb with von Glaserfeld, 1988; Young-Loveridge, 1999). Wright (1998) reworked this material and added aspects of number identification to the framework. The strategy part of the framework considering theories about pupil’s numerical reasoning (Steffe et al. 1983) with refinements from Wright (1998) was further extended, giving teachers five graduated global stages of the structural thinking pupils use. In New Zealand the one framework was split into two (Knowledge and Strategies) and extended to Stage 8 with Stage 0 added. The Numeracy Project recognizes the
interdependence of Knowledge and Strategy and is delivered in the expectation that a pupil’s mathematical understanding is nurtured and developed by carefully maintaining structured instruction in both areas. Teachers in the NDP are encouraged to move away from their traditional instructional style of teaching and teach for relational understanding.

**Composition of SNP**
The essence of the SNP is to promote quality teaching of Mathematics. Classroom lessons should produce deep conceptual rather than procedural understanding. The expectation is that pupils will exhibit flexible and imaginative explanations for the solutions to problems. Considerable attention is given to dimensions of quality teaching. Recent research has shown that quality teaching is fundamental to the improvement of pupils’ outcomes (Alton-Lee, 2003). While readily acknowledging that classrooms are complex, demanding environments, it is possible to identify teacher dispositions and activities that will have a positive impact on pupils achievement: aspirational attitudes have been identified as affective and have been investigated by a number of researchers; the inclusive classroom (Cobb, McClain, & Whiteneck, 1995); focused planning, where lesson content and intent is compiled from a variety of assessment methods and is framed around pupil needs; problem-centered activities, especially those approaches where the expectation is that pupils will be responsible for new knowledge creation (Clarke & Hoon, 2005); high expectations built around a belief that a promotion of higher order thinking skills will motivate pupils to become independent critical learners; connectivist teachers with a propensity and capacity for linking mathematical ideas and vocabulary to actions on materials and an ability to parallel mathematical concepts to realistic contexts (Askew et al, 1997). Research on quality teaching (Shulman, 1987) has identified teachers’ pedagogical and content knowledge as the principle components of quality learning.

Along with research on factors that underpin pupils’ Mathematics education, considerable thought and planning have gone into designing a professional development project that will assist teachers to come to terms with the dimensions of the initiative. The model used to deliver the professional development is structured around an in-school facilitator working with teachers. Schools are invited to participate in the programme, and a contractual agreement is entered into between the schools and the MOE primarily to ensure all staff participate fully. The contract also details the extent of professional development that teachers can expect.

**The Professional Development**
The teacher development model adopted for the introduction of SNP to schools in 2005 differed from that used in other numeracy projects in its use of in-school facilitators (ISF) supported regionally by an external regional facilitator (RF). The RF is charged with supporting the ISF to develop an appreciation of the appropriate pedagogy for complementing the project, especially around investigating ways to present Number Knowledge and Strategy. Further expectations are that the RF will model classroom delivery and will promote and assist with the development of games and activities. These aims are promoted by having regular in-school sessions as well as specific professional development opportunities. Evaluation of this model by Harvey, Higgins & Jackson, (2005) indicated that in general it is successful, particularly in the way that it has impacted on teachers and ISF knowledge, skill and practice. Another benefit was an increased occurrence in professional dialogue within mathematics departments about examining ways for optimizing their school’s Mathematics programme.

Accompanying the professional development programme is a considerable amount of support material. To really facilitate change there needs to be material that teachers can use to assist their lessons and compliment their lesson planning (Fullan 1999). To this end the Ministry initiated the compilation of 8 texts that correspond to the stages in the framework. A web site and publications under the generic heading ‘Figure It Out’ have also been developed to complement teaching programmes. Accompanying these material supports for the project is an understanding that the teaching programmes are to be compiled to address deficiencies in pupils’ Number Knowledge or their familiarity with using Strategies. In essence teachers are to continually assess their pupils, reflect constructively on their next teaching episode and then use the material provided to assist them to plan for the next lesson.
Material Support

The Number Framework is composed of two distinct but complementary sections: the Knowledge component ranks key number items that pupils can recall quickly with minimal effort, and the Strategy section considers the reasoning process by which children use their knowledge to find answers. Both sections are divided into 9 stages which correspond to expected development stages for pupils from the beginning of schooling through to the expectations for a pupil’s Mathematics understanding after 8 years of education. Within the Knowledge framework the aspects identified as important are: number sequencing; grouping and place value; basic facts and written recording. For the Strategy section the 9 stages each demonstrate a progression in levels of sophistication of the rationale for solving problems. The strategies are applied over 3 operation domains: addition and subtraction; multiplication and division; proportion and ratios. The essence of the strategy section is a consideration of a pupil’s progression from being “a counter” to being fluent and competent with multiplication and how they can be extended to thinking proportionally. Both sections contain considerable example detail about specifically which aspects constitute the progression at each stage. Classroom teachers are given detailed professional development about the content and design of the frameworks.

The Diagnostic Interview and the Knowledge Test

These are the assessment tools that teachers use to gain an understanding of where their pupils sit on the number framework and what teaching programme should be organized for them so that they progress through the levels. The diagnostic interview is an opportunity for the teacher to gain an insight into a pupil’s reasoning capability and, more importantly, what aspects of number comprehension the pupil lacks. Up to 24 problems are presented in a non-standard pen and paper format and the pupil are invited to verbalize the process of how he or she has solved problems. Integral to this testing is the teacher’s capacity to illicit responses from the pupil that put more emphasis on the mechanisms about how the answers are formed rather than the answer itself. Teachers detail their answers and consider carefully what the pupil’s replies indicate about where he or she fits on the framework. The Knowledge test is presented as a 10 minute paper and pencil test examining a pupil’s number understanding in the categories: number sequences and order, grouping and place value, basic facts. The results from the two tests enable teachers to set up learning trajectories that address specific gaps in understanding and knowledge. Both tests are repeated at the end of the year enabling progression to be identified. All the results are required to be entered in a secure national database.

Strategy Teaching Model

At the heart of this model is the use of material to validate new concepts, explanations for working of the ideas, for example, demonstrating the addition of fractions or multiplying single place decimals using multi link cubes. Pupils are challenged to demonstrate the answers to a problem by exemplifying it with the cubes. This model requires that from the physical manipulation of the materials pupils will then be able to ‘image’ other solution paths to further problems. After practice the mechanisms they have used will then become part of their general repertoire of solution techniques. This is a fold back model based on the work of numeracy facilitators with Hughes (2002) who in turn has adapted it from P-K theory (Pirie & Kieren, 1994). Fundamental to the teaching model is an expectation that out of the use of material the pupil will construct understanding by moving from the tangible to the imaging after sufficient interaction and then to the generalized understanding. By cycling through this process a number of times and encouraging pupils to reflect on the material, the expectation is that the understanding will be more meaningful and personal. Success with this procedure is related to the extent of teachers’ content knowledge and their ability to model content with materials. Much of the professional development is constructed around an expectation that materials will be used to assist with explanations and that their use will complement and help to verify the solutions to problems.

Outcomes

The SNP numeracy initiative was introduced in 2005 and was intended as a professional development project for secondary school teachers. The SNP initiative is constructed around an acknowledgement that the relationship between Number and Algebra is extensive particularly in the generalized use of strategies. By the end of 2008 approximately 120 secondary schools had been involved in the project.
Results indicated that teachers welcomed the opportunity to establish the extent and depth of pupils’ Number Knowledge and Strategies and were enthusiastic about the teaching model, particularly the use of materials to complement lessons. There was also enthusiasm for putting more emphasis on extolling pupils to construct and articulate their own answers, rather than rely on explanations learnt directly from the teacher. Teachers appeared to be more prepared to put ‘sense making’ as the principle objective of their lessons and were questioning the reliance on teaching algorithm. Teachers welcomed the opportunity to negotiate solution strategies with their classes and have indicated a change in their approach to teaching, moving away from whole class instruction to adopting strategies that allow for differentiated learning programmes. The major area of concern often noted in schools is time. Many teachers feel pressured in their attempts to implement the curriculum and time management to address the intent of SNP has become a challenge.

The SNP continues to receive Ministry support. Measuring significant change is difficult as initially the primary aim of this project was to upgrade teacher pedagogical practices. Lately more emphasis and expectation has been placed on gauging the change in pupils’ capacity to manipulate the Number and Algebra component of the curriculum. Some tension has emerged with a desire to see the expectation for rationalizing answers to problems reflected in the external exams that pupils sit in their last 3 years of schooling, particularly the Number sections

Conclusions

Inviting secondary teachers to examine and analyse their teaching practice and to then reflect on approaches that can have more positive outcomes for pupils is a challenge. The success of SNP is reliant on teachers recognizing that this approach will allow the Mathematics education of their pupils to be more meaningful and their transition to learning Algebra will be less cumbersome or problematic. This, along with an emphasis on teaching programmes that address individual pupil needs, has given the initiative considerable prominence in secondary schools throughout the country. The evolution of the SNP has involved the synthesis of ongoing research, data gathering and analysis, exchanging and sharing best practice, cultivating and examining schools that have successfully incorporated the initiative. Around half of the secondary schools in New Zealand have taken part in the initiative, giving a critical mass of practitioners who are able to critically reflect on the benefits of this approach to teaching Mathematics. The outlook is positive but there are a number of challenges that require further analysis and refinement, in particular, the implementation of the teaching model for the aspiration of catering for individual needs. The transition from Number to Algebra is not as seamless and smooth as has been purported, and there is still much work needed to assist pupils who have difficulty gaining traction with the approaches offered by this initiative. Preparing resources to complement lessons is seen as a demanding and difficult aspect of fulfilling the SNP aspirations. Finally, the success rates of Maori and Pasifika pupils in New Zealand schools is an ongoing concern.

Fundamental to this initiative was a desire to change the way that Mathematics was taught. Behind the notion that ‘mimicking is not teaching’ lies a big challenge, especially for secondary school Mathematics teachers who for the most part considered that their role in a pupil’s Mathematics education was to instruct them in how to apply the most suitable algorithm to a particular problem. In stark contrast SNP is more about opening up the rich Mathematical world that lies behind much of what we teach. It is this Mathematics which runs parallel to what is taught but is often obscured that is one of the most tantalizing and satisfying aspects of the initiative. Having been involved for the past 5 years with this project my reflections upon its value, potential and overall benefits are considered from my experience of teaching Mathematics in secondary schools for 28 years. There are a number of approaches used in SNP for rationalising concepts that have interesting implications for the remainder of a pupil’s Mathematics education at secondary school. Any initiative to alter pedagogical practice is a challenge but SNP brings a framework that complements the whole logical imperative that we associate with this discipline.

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References