USING A MODELLING TASK TO ELICIT REASONING ABOUT DATA
Helena Wessels
Research Unit for Mathematics Education
Stellenbosch University, South Africa
hwessels@sun.ac.za

Abstract
Completing modelling tasks not only develops prospective teachers’ mathematical knowledge and problem solving competencies, but also prepares them to implement mathematical modelling later in their own practice. I present the preliminary findings of one investigation in a longitudinal project in South Africa where 188 prospective teachers completed a modelling task requiring reasoning about data. Responses show a variety of models on different levels of sophistication. Analysis has not been completed yet.

Introduction and theoretical background
Mathematical modelling is increasingly recognised as feasible teaching and learning perspective in schools (Mousoulides, 2009; Wessels, 2009). Mathematical modelling means “applying mathematics to realistic, open problems” (Maaß & Gurlitt, 2009) and engages students in open, non-routine problems that elicit powerful mathematical models which are extended and refined into systems that can be generalised for use in other contexts (Lesh & Doerr, 2003). The mathematical modelling process entails a number of steps that students iteratively go through, usually jumping between the different stages in a non-cyclic way (Arlebäck & Bergsten, 2007). Each of these phases are also characterised by multiple cycles of “interpretations, descriptions, conjectures, explanations and justifications that are iteratively refined and re-constructed by the learner, ordinarily interacting with other learners” (Doerr & English, 2001). Mousoulides (2009) describes six processes in the solving of modelling problems: the understanding of the specified task with its constraints and alternatives; the identification of the relevant constraints; exploring and representing possible alternatives; choosing between alternatives; evaluating the choice; and communicating and defending the decision. The reality of the real-world problem is therefore progressively cut away to convert the real-world problem into a mathematical problem. The mathematical solution is then in the end evaluated against its usefulness in the reality of the situation. Modelling problems require students to make sense of the situation and use quantities and operations that they understand and find useful (Doerr & English, 2001). Students’ exposure to mathematical modelling tasks cannot be a once off experience – they need multiple experiences to explore mathematical constructs and to use their models in new contexts to be able to generalize it.

Open, non-routine or model-eliciting problems (MEA’s) provide the opportunity for students to access and process in the classroom complex mathematics problems at different levels of intellectual sophistication and solve these problems through the interaction between their informal and more formal mathematical knowledge. The paradigm shift from traditional teaching and learning of mathematics to a problem centred approach and a mathematical modelling perspective represents a shift to a more equitable situation in mathematics education. Usable models can originate from solution strategies on different levels, affording achieving students, non-achieving students and students from disadvantaged school environments the same opportunity to build successful models. Students who are exposed to MEA’s often change their beliefs about mathematics positively and enjoy these activities, resulting in a shift to positive dispositions.
International reform movements in mathematics have shown that mathematical modelling may be an effective instrument in bringing about change. We know from research (Clarke, Breed & Fraser 2004; Kim, 2005; Riordan & Noyce, 2001; Schoen, 1993) that mathematical modelling students do at least as well, and often better, on standardised tests; are more able to transfer mathematical ideas into real world; are more confident in mathematics; value communication in mathematical learning more highly than students in conventional classes; and, developed a more positive view about the nature of mathematics than their counterparts. A mathematical modelling perspective therefore brings us a step nearer to turning the dreams of children, parents and teachers into reality where all can achieve in and enjoy mathematics. The use of mathematical modelling tasks in teacher education affords prospective teachers the opportunity to learn worthwhile mathematics while they develop their ability to apply the mathematics they already know in the development of powerful mathematical constructs (Niss, Blum & Galbraith, 2007). Prospective teachers are at the same time prepared to implement mathematical modelling in their future practice. The extension of a mathematics modelling perspective to the education of prospective Foundation Phase (K-3) teachers is not common. In traditional classrooms the focus is on what the teacher teaches and not necessarily on what children understand. Problem solving abilities are usually also not focused on in the early years. A modelling perspective in the Foundation Phase (FP) represents another paradigm shift as it emphasises the importance of mathematics education that fosters understanding as well as the development of problem solving abilities in the primary years.

To be able to implement a modelling perspective in the classroom, it is crucial that prospective teachers are exposed to mathematical modelling in their own education at undergraduate level (Garcia, Maas & Wake, 2009). Successful implementation of a mathematical modelling perspective further depends on teachers

- being familiar with the key concepts of mathematical modelling;
- having appropriate beliefs about the nature of mathematics education; and
- being aware of their own competency to implement this perspective in practice (Maas & Gurlitt, 2009).

Prospective teachers need to be made aware of the nature and spectrum of modelling competencies and how they are used in problem solving. Modelling competencies involves cognitive, metacognitive and affective competencies which are applied in an integrated way in the solving of mathematical modelling tasks (Biccard, 2010).

**Method**

The investigation described here is part of a longitudinal research project to prepare prospective FP teachers to implement a mathematical modelling perspective in their classrooms when they start teaching; to determine whether and how they implement this approach; and what the reasons and challenges are for not implementing MEA’s. This paper reports on the modelling cycles prospective teachers went through and the models they created while solving the second of the MEA’s they were exposed to during the project. Hundred and eighty eight prospective Foundation Phase teachers completed the MEA as part of their mathematics education module. Second, third and fourth year undergraduate prospective teachers (n= 88, 75, 25 respectively) solved the modelling task in groups of two to six in class during 3 to 4 lectures (three to four hours).

The task, “Making Money” (Lesh, Amit & Schorr, 1997), is about an entrepreneur employing nine vendors to sell popcorn and drinks in an amusement park during the summer months. She has to cut the number of vendors employed and asks for recommendations of which six
vendors she should rehire full-time and part-time for the next summer. She supplies tables showing the number of hours worked and the money earned by each vendor when business was busy (high attendance); steady and slow (low attendance). The task is to make recommendations of who she should rehire in a letter, describing in detail how the vendors were evaluated and giving clear explanations so that she can decide whether the method is a good one for her to use.

The task met all six requirements for a MEA, which differs radically from traditional textbook word problems (Lesh, Amit & Schorr, 1997):

- The reality principle: the task focused on a ‘real’ problem that needs to be addressed – not a contrived cleaned-up school textbook problem
- The model construction principle: the task required the construction of a model and the description of assumptions and conditions in the justification of decisions made while constructing the model
- The self-evaluation principle: the task explicitly stated for what purpose and by whom the results were needed to enable the student teachers to evaluate their own solutions and decide whether it needed improvement
- The model-documentation principle: the task required explicit explanations of their thinking about the situation and the solution paths they followed
- The model generalisation principle: the model(s) created could be adjusted and applied to other situations and contexts
- The simple prototype principle: the problem was designed to elicit the creation of a model while still being as simple as possible.

Mathematical modelling essentially involves group work as “... competencies of the group are likely to be greater than those of individuals” (Biccard, 2010; Mousoulides, 2009; Zawojewski, Lesh & English, 2003). Individuals verbalise and criticise ideas more spontaneously in a small group setting, resulting in a higher level of creativity and the development of more sophisticated systems.

Data sources included the researcher’s field notes, student teachers’ rough notes, mind maps and workings as well as their final reports describing their recommendations, their solutions paths and reasons for specific decisions made along the way. The analysis is still in progress. The project is being conducted with problem centred teaching and learning as context – the researcher did not suggest any strategies or solution paths, neither did she prompt student teachers to use the different conceptual models. She provided resources and gave ample time for students to complete the activity. They were allowed to use technology such as pocket calculators and computers.

When student teachers in the project had to solve a MEA for the first time, they could not believe that they were not going to be told how to solve the problem as was the case in school mathematics. Some groups only started working productively on the task after almost an hour and most students took 3 to 4 hours to complete the task, some even took longer. With the second MEA students did not start right away working on the task, but this time they knew that they had to do it on their own and that the researcher would not give hints or prompt strategies and solutions. By the time they had to solve the third MEA, they immediately started working on the problem and most groups finished after about two and a half hours.

**Preliminary findings**
Responses fall into two big categories (See Table 1). In the first category models were developed without combining the two data sets, i.e. by totalling earnings and totalling number of hours worked (n=6) and focusing on averages (N=1). Two groups calculated only totals for
income and for hours worked, while 4 groups totalled income an hours worked for the different shifts (busy, steady and slow times) and ranked the vendors accordingly. One group used average income and average hours worked to develop their model. In the second category the data set with number of hours worked was combined with the data set with earnings to calculate rand-per hour (n=41). This category showed 12 different interpretations of the problem (see Table 1), with groups inventing decision-making rules on different levels of sophistication.

**Table 1: Analysis of responses**

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Number of group responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>No combining of the two data sets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total income and total number of hours worked</td>
<td>Totals only</td>
<td>2</td>
</tr>
<tr>
<td>N=6</td>
<td>Totals for different shifts, &amp; rankings</td>
<td>4</td>
</tr>
<tr>
<td>Average income and average number of hours</td>
<td>Average hours &amp; average income</td>
<td>1</td>
</tr>
<tr>
<td>worked N=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Combining the two data sets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income per hour (R/h)</td>
<td>R/h for months separately</td>
<td>3</td>
</tr>
<tr>
<td>N=41</td>
<td>R/h over 3 months</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>R/h for different shifts &amp; R/h overall</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>R/h for different shifts &amp; R/h overall &amp; ranking</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>R/h for different shifts &amp; R/h overall &amp; ranking using ratio</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>R/h &amp; consistent high scores for different shifts</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>R/h &amp; R/h for slow times</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>R/h &amp; R/h in busy times (for deciding part time vendors)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>R/h &amp; weighting (points system)</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>R/h &amp; % of average R/h of all vendors together</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>R/h &amp; average of all vendors together – deviation from average</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>R/h &amp; average of all vendors together – above this average</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>48</td>
</tr>
</tbody>
</table>

Prospective teachers mostly used tables and bar graphs to represent their interpretations of the task, although a few (inappropriate) broken line graphs and pie graphs were used.

**Discussion**

Realistic problems require working with and understanding different kinds of quantities and operations than in traditional tasks (Doerr & English, 2001). The “Money Making” task entailed statistical analysis and included multiple views and representations of the data (mostly tabular and graphical), measures of central tendency (averages and deviation from averages), as well as the combining of data and analyses of trends. Operations needed in the task therefore differ from operations in traditional problems: additional to adding and dividing quantities and calculating averages, student teachers also needed to sort, organise,
select, combine and transform the entire data set rather than just working with single data points.

Groups had to describe in detail the solution paths they followed in the process of developing the model. They were required to identify different alternatives and evaluate and justify their choices of alternatives. One procedure was to brainstorm together and then trim away less useful suggestions to pursue one or two ideas. Another procedure was that each group member had to come up with an own idea and do the preliminary working out. Ideas were then compared and useful alternatives further developed. A third modus operandi was to first explore just one idea initially and then work towards other alternatives. Group 38 for example used income per hour to identify the best six vendors and then compared income per hour for the slow shifts because “... if one shows that they can bring in the most money in the slow times then they are good vendors. Not everyone enjoys working slow times and this suggests that Thandi, Jose and Maria are willing to work and are more dedicated to their jobs than the rest. They should be hired full-time”. Group 41 ranked vendors by a weighting scheme and remarked: “Using this method, each vendor’s performance can be clearly compared against that of the others, and a fair decision (one that takes into account all factors – the number of hours they worked, how much money they collected, as well as when they worked) can be made”.

Different solution paths for similar interpretations were common. Some groups for example first worked out the average income per hour of each vendor for each shift per month and then added it all together to calculate an overall average for the individual while other groups worked out total earnings and total hours for each individual and then calculated the total average.

Although the prospective teachers did not receive specific instruction in these ideas, some groups developed quite sophisticated models. Group 47 for example combined the two data sets by calculating income per hour per shift of each vendor, but also focused on the average productivity (income per hour) of the whole group. They ranked vendors according to deviations from this average to decide which vendors to recommend for full-time and which to recommend for part-time employment.

Groups who used only totals or averages of income and/or hours did not critically evaluate their solution paths and stuck with one idea. Most other groups explored the usefulness of different models before deciding on a specific one. A few groups compared conclusions reached through the use of earlier models to check the conclusions reached with a later more sophisticated model.

Summary
The models described above were developed without any interference or guidance from the researcher. Most groups went through a number of modelling cycles through which they progressed from focusing on subsets or isolated pieces of the data to considering combined data sets and underlying trends and regularities.

References


